MultiGreen: Cost-Minimizing Multi-source Datacenter Power Supply with Online Control

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ABSTRACT
Faced by soaring power cost, large footprint of carbon emission and unpredictable power outage, more and more modern Cloud Service Providers (CSPs) begin to mitigate these challenges by equipping their Datacenter Power Supply System (DPSS) with multiple sources: (1) smart grid with time-varying electricity prices, (2) uninterrupted power supply (UPS) of finite capacity, and (3) intermittent green or renewable energy. It remains a significant challenge how to operate among multiple power supply sources in a complementary manner, to deliver reliable energy to datacenter users over time, while minimizing a CSP’s operational cost over the long run. This paper proposes an efficient, online control algorithm for DPSS, called MultiGreen. MultiGreen is based on an innovative two-timescale Lyapunov optimization technique. Without requiring a priori knowledge of system statistics, MultiGreen allows CSPs to make online decisions on purchasing grid energy at two time scales (in the long-term market and in the real-time market), leveraging renewable energy, and opportunistically charging and discharging UPS, in order to fully leverage the available green energy and low electricity prices at times for minimum operational cost. Our detailed analysis and trace-driven simulations based on one-month real-world data have demonstrated the optimality (in terms of the tradeoff between minimization of DPSS operational cost and satisfaction of datacenter availability) and stability (performance guarantee in cases of fluctuating energy demand and supply) of MultiGreen.

1. INTRODUCTION
The proliferation of cloud computing services has promoted massive, geographically distributed datacenters. Cloud service providers (CSPs) are typically facing three major power-related challenges: (1) Skyrocketing power consumption and electricity bills, \textit{e.g.}, Google with over $1,120\text{GW}h$ and $\$67M$, and Microsoft with over $600\text{GW}h$ and $\$36M$ \cite{31}. (2) Serious environmental impact, as IT carbon footprints can occupy 2\% of the global CO\textsubscript{2} emissions reportedly \cite{18}. (3) Unexpected power outages, \textit{e.g.}, Amazon experienced an outage in October 2012 in its US-East-1 region, which was triggered by a series of failures in the power infrastructure \cite{17}.

To address these challenges, modern CSPs begin to equip their datacenter power supply systems (DPSS) with multiple power sources in a complementary manner, as illustrated in Fig. 1. \textit{First}, modern datacenters obtain their primary power from a smart grid. Smart grids typically provide different pricing schemes at different timescales, such as a long-term-ahead pricing market and the real-time market \cite{1,15,19,21,37}. \textit{Next}, datacenters are equipped with uninterrupted power supplies (UPSs) to guard against possible power failures. The supply of UPSs may mostly keep a datacenter running for $5 \sim 30$ minutes upon a power outage \cite{36}. \textit{Finally}, UPSs are also starting to green their datacenter operations by integrating on-site renewable energy, such as solar and wind energies \cite{8, 10, 22-24, 35, 41}. The renewable energy is connected to the grid via a grid-tie device, which combines electricity produced from the renewable sources and the grid on the same circuit for power supply \cite{4,10}. The amount of renewable energy produced could vary significantly over time \cite{18}. UPS can be used to store energy during periods of high levels of renewable energy generation and/or low electricity prices in the grid markets. When the renewable energy is insufficient or prices from the grid are high, the UPS battery can be discharged to provide
power [11–13, 20, 36, 38].

An important problem faced by CSPs is how to minimize the long-term cost of running their datacenters. Several key decisions need to be made in an online fashion when operating such a DPSS. (1) How much power to be purchased from the grid’s long-term market and the real-time market, respectively? (2) How to efficiently utilize the available renewable energy? (3) How to opportunistically use the UPS battery to store excess power generated/purchased and supply power when needed? It is challenging to optimally utilize the multiple sources to reliably power a datacenter, while minimizing its operational cost. On the demand side, power demand in a datacenter is time-varying, due to variant resource usage of different applications running in the datacenter; on energy supply side, the grid may offer long-term prices and real-time prices, which change over time; further, the unpredictable nature of renewable energy adds onto the supply uncertainties.

There have been a number of works investigating datacenter power supply in cases of varying power demand, renewable energy supply and electricity prices from smart grids. These work may either assume a priori knowledge of the power demand [1, 19, 20, 37], or require a substantial amount of statistics of the system dynamics, in order to predict the future demand based on different forecast techniques [10, 15, 21, 41]. Some only optimize single-day or single-household power supply [14, 21], while others do not consider the interactions among renewable energy usage, multi-timescale grid power purchasing and energy storage from the prospective of a datacenter operator [14, 19, 20, 27, 29, 30, 33]. In contrast, we seek to design an efficient online control strategy for long-term cost minimizing operation of the DPSS under dynamic power demand and uncertain renewable energy supply in a synergetic manner, without requiring a priori knowledge or stationary distribution of system statistics.

Specifically, based on a stochastic optimization model that characterizes time-varying power demand and renewable energy supply, finite UPS capacity and two-timescale grid markets, we derive an online DPSS control algorithm –MultiGreen – by applying a two-stage Lyapunov optimization technique [9, 26, 39]. MultiGreen decides the amount of energy to purchase from the grid’s long-term market in intervals of longer periods of time, as the basic energy supply to address demand dynamics and real-time price fluctuations in the future interval: MultiGreen also decides the amount of energy to purchase from the real-time market, as well as the amount of energy to store into or discharge from the UPS batteries, in shorter time scales. The online decisions are set to best utilize the available renewable energy produced over time and the periods with low electricity prices, in order to minimize the operational cost in the long run of the datacenter.

A salient feature of MultiGreen is that, even without requiring any a priori knowledge of the system dynamics, it can arbitrarily approach the optimal offline cost which is computed with full knowledge of the system over its long run within a provable $O(1/V)$ gap. The algorithmic parameter $V$ serves as a control knob, by adjusting which CSPs can control the tradeoff between the minimization of the DPSS operational cost and satisfaction of the constraints of datacenter availability and UPS lifetime. We analyze the performance of our online control algorithm with rigorous theoretical analysis. Further, we demonstrate the optimality (in terms of the tradeoff between minimization of DPSS operational cost and satisfaction of datacenter availability constraint) and stability (performance guarantee in cases of fluctuating power demand and supply) achieved by MultiGreen, using extensive simulations based on one-month worth of traces from live power systems.

2. SYSTEM MODEL AND OBJECTIVE

As illustrated in Fig. 2, we consider a DPSS system operating in a discrete-time model. Time is divided into $K (K \in \mathbb{N}_+)$ coarse-grained slots of length $T$ in accordance with the interval length of grid’s long-term market, e.g., days or hours [21]. Each coarse-grained time slot is further divided into $N_T (N_T \in \mathbb{N}_+)$ fine-grained time slots, e.g., $N_T = 5$ in Fig. 2. Empirically, each fine-grained time slot is 15 or 60 minutes long per which the datacenter can adjust power control strategies in a more prompt fashion [19, 39].

2.1 Online Control Decisions

We assume that datacenter energy demand $d(t)$ and renewable energy generation $g(t)$ are random variables. We don’t assume they follow any specific probability distribution functions. We assume a datacenter provider can buy electricity from the grid’s long-term contracts or buy electricity in spot markets $. The operation of DPSS includes four key control decisions in two timescales:

*This is a realistic assumption. In fact, Google formed a Google Energy LLC, and get approval to buy and even sell electricity 3 years ago [16]. In the future, more datacenter...
2.1.1 Long-term-ahead Energy Purchase

At the beginning (the first fine-grained time slot) of each coarse-grained time slot \( t = kT(k = 1, 2, ..., K) \), the DPSS observes the energy demand \( d(t) \) and renewable energy \( g(t) \) generated during time slot \( t \). Then, the DPSS makes a decision on how much energy \( y_{ref}(t) \) to be purchased at a price \( p_{lt}(t) \) (with an upper bound price \( P_{\max} \)) in the long-term market. The DPSS averagely schedules energy \( y_{lt}(t) = y_{ref}(t)/N_T \) to be used in each fine-grained time slot in this coarse-grained time slot. For example, if the DPSS decides to purchase 100KW in the day-ahead grid market according to the observation of the current demand and renewable energy production, it will schedule 20KW for each fine-grained time slot in the next day when \( N_T = 5 \).

2.1.2 Real-time Energy Balancing

Since the primary costs for renewable energy generation are construction costs such as deploying solar panels and wind turbines, their operational cost is negligible [35], and we focus on operational cost minimization in this paper. Thus, the renewable energy is assumed to be harvested free after deployment, and we preferentially use it. When the renewable energy is generated during time slot \( t \), we use it to meet energy demand as much as possible. If there is excess renewable energy, we use the battery to store it.

Specifically, at each fine-grained time slot \( \tau \in [t, t + T - 1] \), the actual energy demand \( d(\tau) \) and available renewable energy \( g(\tau) \) can be readily observed by the DPSS. If the long-term-ahead purchasing and the renewable energy are enough to meet the current energy demand, i.e., \( y_{lt}(\tau) + g(\tau) \geq d(\tau) \), then no real-time energy discharging/purchasing is needed. Otherwise, the DPSS has to make a decision on whether to discharge energy \( D(\beta(\tau)) \) from the battery. If the discharged power is still not enough, the DPSS decides how much additional energy \( y_{rt}(\tau) \) to be purchased from the grid’s real-time market at the real-time price \( p_{rt}(\tau) \leq P_{\max} \) to fulfill the current demand. Any superfluous energy is used to charge the battery \( R(\tau) \). Thus, we have:

\[
y_{lt}(\tau) + y_{rt}(\tau) + D(\beta(\tau)) + g(\tau) - R(\tau) = d(\tau),
\]

where \( D(\beta(\tau)) \) denotes the amount of UPS energy discharged at the depth-of-discharge (DoD) level of \( \beta(\tau) \) \( (\beta(\tau) \in [0, 1]) \). DoD is a measure of how much energy has been withdrawn from the battery, expressed as a percentage of the full discharging capacity. For example, let \( D_{\max} \) denote the maximum energy that we can discharge per time, then \( D(\beta(\tau)) = \beta(\tau)D_{\max} \). The battery is either charged or discharged or not in use at each time slot, i.e., \( R(\tau) \cdot D(\beta(\tau)) = 0 \).

2.2 Online Control Constraints

There are a series of constraints that the above decision-making should satisfy:

2.2.1 Balancing procurement accuracy and cost

In practice, the price of electricity in the grid’s real-time market tends to be higher on average than that in the grid’s long-term market, i.e., \( \mathbb{E}p_{rt}(\tau) > \mathbb{E}p_{lt}(t) \) [1, 21, 37], as upfront payment is associated with cheaper contract prices in the long-term market. Hence, when procuring energy in the two-timescale markets, the DPSS should make the best tradeoff between procurement accuracy and power cost. Additionally, we assume that the maximal amount of power that the datacenter can draw from the grid at each time is limited by \( P_{\text{grid}} \):

\[
0 \leq y_{lt}(\tau) + y_{rt}(\tau) \leq P_{\text{grid}}.
\]

2.2.2 Guaranteeing datacenter availability

Let \( m(\tau) \) denote the UPS energy level at time \( \tau \). We assume that the efficiencies of UPS charging and discharging are the same, denoted by \( \eta \in [0, 1] \), e.g., \( \eta = 0.8 \) means that only 80% of the charged or discharged energy is useful when charging or discharging. The dynamics of UPS level \( m(\tau) \) can be expressed as:

\[
m(\tau + 1) = m(\tau) + \eta R(\tau) - D(\beta(\tau))/\eta.
\]

We assume that under any feasible control algorithm, the UPS battery always reserves a minimum energy level \( M_{\min} \), to guarantee reliable datacenter operation in case of power outage. For instance, the energy buffer eBuff [11] always retains five-minute-worth of reserved energy in UPS to ensure datacenter availability. We assume that UPS has a capacity of \( M_{\text{UPS}} \), thus we have:

\[
M_{\min} \leq m(\tau) \leq M_{\text{UPS}}.
\]

Typically, \( M_{\min} \) can power the peak demand of a datacenter for about a minute, while \( M_{\text{UPS}} \) can for 5 ~ 30 minutes [36].

2.2.3 UPS lifetime and operational cost

In practice, UPS is constrained by the maximum amounts of energy for recharge and discharge per time:

\[
0 \leq D(\beta(\tau)) \leq D_{\max}, 0 \leq R(\tau) \leq R_{\max},
\]

where \( D_{\max} \) and \( R_{\max} \) are the maximum energy that we can recharge and discharge UPS per time, respectively.

It has been practically shown that the UPS lifetime is a decreasing function of DoD and charge/discharge cycles [11]. The cost of operating the battery is a function of how often/much it is charged and discharged. We assume that the costs of each recharging and discharging operation are the same, denoted as \( C_r \). If a new UPS costs \( C_{\text{ups}} \) to purchase and it can sustain \( L_{\text{ups}} \) total cycles of charging and discharging at the maximum DoD, then \( C_r = C_{\text{ups}}/L_{\text{ups}} \). If the lifetime of the
UPS is $Life_{ups}$, then the maximum allowable discharging and charging number over the long run $[0, t-1]$ where $t \in KT$ is $N_{\text{max}} = L_{ups} \cdot \frac{KT}{Life_{ups}}$. $N_{\text{max}}$ satisfies:

$$0 \leq \sum_{\tau=0}^{t-1} a(\tau) \leq N_{\text{max}},$$

where $a(\tau)$ denotes whether the UPS is used in time slot $\tau$ or not, that is $a(\tau) = 1$ if $D(\tau) > 0$ or $R(\tau) > 0$, $a(\tau) = 0$ otherwise. Hence, at time slot $t$, the operational cost of UPS operation is $a(t)C_r$.

### 2.3 Stochastic Cost Minimization Formulation

At each fine-grained time slot $\tau$, the DPSS operational cost is the sum of costs for grid energy purchasing and UPS charging/discharging. Therefore, $Cost(\tau) \triangleq y_{rt}(t)p_{rt}(t) + y_{rt}(\tau)p_{rt}(\tau) + a(\tau)C_r$. We seek to design an online DPSS control algorithm that can systematically make online decisions by solving the following stochastic cost minimization problem $P1$:

$$\min_{y_{rt}, D(\beta), R} \text{Cost}_{av} \triangleq \liminf_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[Cost(\tau)] \text{ (7)}$$

s.t. $\forall t$ : constraints (4)(5)(6).

Since the battery can be charged to store energy or discharged to serve demand, the current control decisions are coupled with the future decisions. For example, current decisions may leave insufficient battery capacity for storing renewable energy produced in the near future, or overuse the battery and threaten datacenter availability. To solve this optimization problem, the commonly used dynamic programming technique and Markov decision process suffer from a curse of dimensionality and require significant knowledge of the demand and supply over the long term [2, 15]. In contrast, the recently developed Lyapunov optimization framework [9, 26] is shown to enable the design of online control algorithms for such constrained optimization of time-varying systems without requiring a priori knowledge of the future workload and costs. In particular, our above two-timescale power delivery model fits well the two-stage Lyapunov optimization framework [39], that enables us to perform two levels of control strategies at two levels of time granularity. Therefore, we design our online control algorithm based on the two-timescale Lyapunov optimization technique.

### 2.4 An Optimal Offline Algorithm

Now we present a polynomial-time optimal offline solution for problem $P1$ as a benchmark for comparison. In the theoretically optimal scenario, DPSS knows all future system statistics including energy demand $d(t)$, renewable energy production $g(t)$ and grid energy prices $p_{rt}(t), p_{rt}(\tau), \forall t \in [0, 1, ..., KT]$. First, we present the following straightforward Lemma 1 about the optimal real-time energy purchasing without proof for brevity.

**Lemma 1.** In every optimal solution of the optimization problem $P1$, it holds $\forall t$, $y_{rt}(\tau) = 0$ or $p_{rt}(\tau) = 0$.

The above lemma implies that real-time energy purchasing is unnecessary in the optimal solution, where all the future statistics are known in advance. Thus, solving the optimization problem $P1$ is equivalent to solving $K$ single time-slot problems $P2$ as follows, $\forall t \in [0, 1, ..., KT]$, at the first fine-grained time slot of each coarse-grained time slot over the long horizon $KT$:

$$\min_{y_{rt}, D(\beta), R} \quad y_{rt}(t)p_{rt}(t) + a(t)C_r$$

s.t. $y_{rt}(t) + y_{rt}(\tau)p_{rt}(\tau) + a(\tau)C_r$.

$\forall t$ : constraints (4)(5)(6).

$P2$ only includes linear terms, and hence can be solved in polynomial time using standard linear programming techniques, e.g., interior point methods [3].

### 3. Online Algorithm Design

Now we develop an online algorithm to achieve near-optimal solution without a priori knowledge of power demand and renewable energy generation.

#### 3.1 A Lyapunov Optimization Solution

To guarantee datacenter availability and deliver reliable energy to datacenter power demand, we should guarantee constraint (4) of battery level. We use an auxiliary variable $X(t)$ to track the battery level, defined as follows:

$$X(t) = m(t) - VP_{max}/T - M_{min} - D_{max}/\eta,$$

where $V \geq 0$ is a control parameter to be specified later, which affects the distance to the optimal value and is related to the battery capacity. The intuition behind $X(t)$ is that by carefully tuning the weights $V$ for decision-making, we can ensure that whenever charging/discharging the battery, the energy level in the battery always lies in the feasible region $[M_{min}, M_{UPS}]$. Recall that $m(t)$ is the actual battery level in time slot $t$ and evolves according to Eq. (3). The dynamics of $X(t)$ is given as:

$$X(t+1) = X(t) + \eta R(\tau) - D(\beta(\tau))/\eta.$$

In Theorem 2, we will prove that for any time slot $t$, the battery level $m(t)$ is always in the safe range, since $X(t)$ is deterministically lower and upper bounded.

With the queue, we transform the inequality constraint (4) into a queue stability problem [26]. As a scalar measure of the queue length, we define a quadratic Lyapunov function as:

$$L(t) \triangleq \frac{1}{2} X^2(t).$$

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This represents a scalar metric of queue congestion in the system. To keep the system stable by persistently pushing the Lyapunov function towards a lower congestion state, we introduce the $T$-slot conditional Lyapunov drift as:

$$
\Delta_T (t) \triangleq \mathbb{E}[L(t+T) - L(t)|X(t)].
$$

Then following the Lyapunov drift-plus-penalty framework [9], we add a function of the expected operational cost over $T$ slots (i.e., the penalty function) to (12) to obtain the drift-plus-penalty term. Our control algorithm is designed to make decisions on $\gamma_{be}(t), \gamma_{ct}(t)$, and $\beta(t)$ and $R(t)$ to minimize an upper bound on the following drift-plus-penalty term in every time frame of length $T$:

$$
\Delta_T (t) + \mathbb{E}\{ \sum_{t=t}^{t+T-1} \text{Cost}(\tau)|X(t)\},
$$

where the control parameter $V$ is chosen by the CSP to tune the tradeoff between DPSS cost minimization and datacenter availability (battery level). For instance, if $V$ is set to be larger and more emphasis is put to cost minimization, then UPS will be overly used for certain times and thus datacenter availability only achieves a weak satisfaction. A key derivation step is to obtain an upper bound on this term. The following Theorem 1 gives the analytical bound on the drift-plus-penalty term.

**Theorem 1. (Drift-plus-Penalty Bound)** Let $V > 0$ and $t = kT (k \in \mathbb{Z}_+)$, i.e., an upper bound of the drift-plus-penalty framework in (13). However, to minimize the RHS of (14), the DPSS needs to know the concatenated queue backlog $X(t)$ over future time frame $\tau \in [t, t + T - 1]$. The queue $X(t)$ depends on UPS battery level $m(t)$, the energy demand $d(t)$ and available renewable energy $g(t)$. The highly variable nature of energy demand, renewable energy and electricity prices has been a major obstacle to make accurate decisions. In practice, system operators can use forecast techniques to predict the future statistics. However, the $90^{th}$ percentile forecast error for 1-hour-ahead prediction of the renewable energy can be as high as 22.2% [10]. For a 25% penetration of wind energy in a smart grid, the day-ahead forecast error of wind energy generation can result in an additional operational cost of $4.41 per MWh for the operator [40].

Therefore, we instead approximate near-future queue backlog statistics using the current values, i.e., $X(t) = X(t)$ for $t < \tau \leq t + T - 1$. This significantly reduces the computational complexity and eliminates the need for any forecast technique in our algorithm, while only bringing a slight “loosening” of the upper bound on the drift-plus-penalty term, as proved in Corollary 1. For this approximation, we will show that our algorithm can still approach the optimal performance with a proven bound in Theorem 3 in Sec. 4 and simulations in Sec. 5.3.

**Corollary 1. (Loosening Drift-plus-Penalty Bound)** Let $V > 0$ and $t = kT$ for some nonnegative integer $k$. 

#### Remark:
Our control algorithm is then constructed following the “minimizing drift-plus-penalty” principle of the Lyapunov optimization technique: at every time slot, choose a set of feasible energy purchasing and UPS battery charging/discharging actions to minimize the right-hand-side (RHS) of (14). The parameter $V$ is chosen to enforce different weights to time-averaged operational cost $Cost_{av}$ and queue drift $\Delta_T(t)$ for the CSP to tune the tradeoff between DPSS cost minimization and datacenter availability. The operational cost achieved can be smaller if datacenter availability is just weakly satisfied, e.g., slightly overcharge the UPS battery.

### 3.2 Relaxed Optimization Problem

The key principle of Lyapunov optimization framework is to choose online control policies to minimize the right-hand-side (RHS) of (14) in Theorem 1, i.e., an upper bound of the drift-plus-penalty framework in (13). However, to minimize the RHS of (14), the DPSS needs to know the concatenated queue backlog $X(t)$ over future time frame $\tau \in [t, t + T - 1]$. The queue $X(t)$ depends on UPS battery level $m(t)$, the energy demand $d(t)$ and available renewable energy $g(t)$. The highly variable nature of energy demand, renewable energy and electricity prices has been a major obstacle to make accurate decisions. In practice, system operators can use forecast techniques to predict the future statistics. However, the $90^{th}$ percentile forecast error for 1-hour-ahead prediction of the renewable energy can be as high as 22.2% [10]. For a 25% penetration of wind energy in a smart grid, the day-ahead forecast error of wind energy generation can result in an additional operational cost of $4.41 per MWh for the operator [40].

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Replacing the concatenated queue $X(\tau)$ with $X(t)$, the drift-plus-penalty satisfies:

$$\Delta_T (t) + V E \{ \sum_{\tau=t}^{t+T-1} C o s t(\tau) | X(t) \} \leq B_2 T + E \{ \sum_{\tau=t}^{t+T-1} X(t) [ R(\tau) \eta - D(\beta(\tau))/\eta] | X(t) \}$$

where $B_2 = B_1 + T(T - 1) [ R_{\text{max}}^2 \eta^2 - D_{\text{max}}^2 \eta^2 ]/2$, and $B_1$ is given in Theorem 1.

**Proof.** According to Eq. (10), for any $\tau \in [t, t + T - 1]$, we can get that:

$$X(t) - (\tau - t) D_{\text{max}}/\eta \leq X(\tau) \leq X(t) + (\tau - t) \eta R_{\text{max}}.$$

Therefore, recalling each term in Eq. (14), we have:

$$\sum_{\tau=t}^{t+T-1} X(\tau) [ R(\tau) \eta - D(\beta(\tau))/\eta] \leq \sum_{\tau=t}^{t+T-1} [ X(t) + (\tau - t) \eta R_{\text{max}}] R(\tau) \eta$$

$$= \sum_{\tau=t}^{t+T-1} X(t) [ R(\tau) \eta - D(\beta(\tau))/\eta]$$

$$+ \sum_{\tau=t}^{t+T-1} (\tau - t) [R_{\text{max}} R(\tau) \eta^2 - D_{\text{max}} D(\beta(\tau))/\eta^2]$$

$$\leq \sum_{\tau=t}^{t+T-1} X(t) [ R(\tau) \eta - D(\beta(\tau))/\eta]$$

$$+ T(T - 1) [ R_{\text{max}}^2 \eta^2 - D_{\text{max}}^2 \eta^2 ]/2.$$

Therefore, by defining $B_2 = B_1 + T(T - 1) [ R_{\text{max}}^2 \eta^2 - D_{\text{max}}^2 \eta^2 ]/2$, substituting the above inequality into (14), we prove the theorem. □

### 3.3 Two-timescale Online Control Algorithm

Comparing RHS of (14) with RHS of (15), we can see that the RHS of (15) gives a larger upper bound than the RHS of (14). We seek to minimize the RHS of (15), to derive the online decisions. The control decision $y_{t+1}(t)$ should be made at the beginning of each coarse-grained time slot while $y_{t+1}(\tau), D(\beta(\tau)),$ and $R(\tau)$ are made at each fine-grained time slot. Thus, we can separate the problem into two independent subproblems $P_3$ and $P_4$ as given in our MultiGreen Algorithm 1, to make decisions in the two timescales, respectively. At each coarse-grained time slot $t = kT$, MultiGreen decides how much energy $y_{t+1}(T) = N_T y_{t+1}(t)$ to be purchased from the grid’s long-term energy market. The decision should make sure that the current energy demand is met and the battery stores enough energy for the future need. At each real-time slot $\tau \in [t, t + T - 1]$, MultiGreen decides real-time market procurement $y_{t+1}(\tau)$, and UPS battery discharging $D(\beta(\tau))$ and charging $R(\tau)$ to supply energy when needed or store additional energy, so as to match the power demand and supply. At the end of each time slot, MultiGreen updates its queue statistics.

**Algorithm 1:** The Online Algorithm MultiGreen.

1) **Long-term-ahead Energy Planning:** At each coarse-grained time slot $t = kT (k \in \mathbb{Z}^+)$, the DPSS decides the optimal power procurement $y_{t+1}(t)$ in the grid’s long-term market to minimize the following problem $P_3$:

$$\min \ E \{ \sum_{\tau=t}^{t+T-1} V [ y_{t+1}(t) p_{t+1}(t) + y_{t+1}(\tau) p_{t+1}(\tau)] | X(t) \}$$

$$\text{s.t.} \ { \sum_{\tau=t}^{t+T-1} X(t) [ R(\tau) \eta - D(\beta(\tau))/\eta] | X(t) \}$$

2) **Real-time Energy Balancing:** Then the DPSS averages the energy $y_{t+1}(t) = y_{t+1}(t)/N_T$ to be used for each fine-grained time slot $\tau \in [t, t + T - 1]$. The DPSS decides real-time energy procurement $y_{t+1}(\tau)$, and UPS battery discharging $D(\beta(\tau))$ and charging $R(\tau)$ to minimize the following optimization problem $P_4$:

$$\min \ V y_{t+1}(\tau) p_{t+1}(\tau) + X(t) [ R(\tau) \eta - D(\beta(\tau))/\eta]$$

$$\text{s.t.} \ (1)(2)(5)(6)$$

3) **Queue Update:** Update the actual and virtual queues using Eq. (3) and Eq. (10).

**Remarks:** MultiGreen is computationally efficient. Each time it only needs to solve two linear programs with four variables ($y_{t+1}(t), y_{t+1}(\tau), D(\beta(\tau)), R(\tau)$) and four linear constraints in (1)(2)(3)(5)(6). We can easily solve the two sub-problems $P_4$ and $P_5$ using classical linear programming approaches, e.g., interior point methods [3]. MultiGreen makes online control decisions $y_{t+1}(t), y_{t+1}(\tau), D(\beta(\tau))$ and $R(\tau)$ solely based on the current available statistics at each time slot, including queue statistics, energy demand, volume of the available renewable energy, energy prices and UPS energy level. These statistics typically only require a few bits to store, and take very little time to calculate and transmit. Besides, interior point methods have a low computational complexity (usually polynomial time) in practice [3]. Though advanced prediction techniques can complement MultiGreen to make more accuracy decisions, a tradeoff exists between the benefits of decision accuracy and complexity of implementing the forecast techniques. Note that, MultiGreen is more suitable for delay-sensitive energy demand than delay-tolerant demand. That is, MultiGreen seeks to address energy demand when it is generated immediately. No energy demand should be delayed to a future time to address. We leave it as the future work to design a smart power supply system for mixed workloads.
4. PERFORMANCE ANALYSIS

In this section, we analyze our MultiGreen algorithm in terms of performance bound and robustness.

4.1 Performance Bound

We first analyze the gap between the result achieved by MultiGreen, if accurate knowledge of \(X(\tau)\) in the future coarse-grained interval is employed rather than our approximation. We assume that the theoretical offline optimal objective function value is \(\phi^{opt}\) of the cost minimization problem \(P1\).

**Theorem 2. (Performance Bound):** The time-averaged cost \(\text{Cost}_{av}^{Green}\) achieved by the MultiGreen algorithm based on accurate knowledge of \(X(\tau)\) in the future coarse-grained interval satisfies the following bound with any given control parameter \(V(V > 0)\):

1. The time-average cost \(\text{Cost}_{av}^{Green}\) achieved by MultiGreen satisfies the following bound:

\[
\text{Cost}_{av}^{Green} \triangleq \lim_{t \to \infty} \inf \int_1^{t-1} \mathbb{E}[\text{Cost}(\tau)] \leq \phi^{opt} + \frac{B_2}{V}; \tag{16}
\]

where \(B_2\) is given in Corollary 1.

2. The UPS battery level \(m(t)\) is always in the range \([M_{min}, M_{UPS}]\). Datacenter availability is satisfied.

3. All control decisions are feasible.

**Proof.** (1) Let \(t = KT(k \in \mathbb{Z}_+\) and \(\tau \in [t, t+T-1]\). From the optimal offline policy in Sec. 2.4, we know that there is an optimal solution \(\phi^{opt}\). Since MultiGreen minimizes the RHS of Eq. (15), plugging the policy \(\pi\) into the RHS of Eq. (15), we have:

\[
\Delta(t) + V\mathbb{E}\left\{\sum_{\tau=t}^{t+T-1} \text{Cost}_{av}^{Green}(\tau)\right\} X(\tau) \leq B_2 T + V\phi^{opt}.
\]

Taking the expectation of both sides and rearranging the terms, we get:

\[
\mathbb{E}\left\{L(t+T) - L(t)\right\} + V T \mathbb{E}\left\{\text{Cost}_{av}^{Green}(t)\right\} \leq B_2 T + V T \phi^{opt}.
\]

Summing the above over \(t = KT, k = 0, 1, 2, ...K - 1\), using the fact that \(L(t) > 0\), and dividing both sides by \(V KT\), we obtain:

\[
\frac{1}{KT} \mathbb{E}\left\{\sum_{\tau=0}^{KT-1} \text{Cost}_{av}^{Green}(\tau)\right\} \leq \phi^{opt} + \frac{B_2}{V}.
\]

Taking the limit as \(K \to \infty\), we complete the proof.

(2) We first observe that subproblem \(P4\) has the following properties related to battery operation:

**Lemma 2.** If \(X(t) > 0\), then \(R(t) = 0\); if \(X(t) < -V_{\max}/T\), then \(D(\beta(t)) = 0\).

We first prove that \(-V_{\max}/T - D_{\max}/\eta \leq X(t) \leq M_{UPS} - V_{\max}/T - M_{\min} - D_{\max}/\eta\). We prove the result using induction. Since \(X(0) = m(0) - V_{\max}/T - M_{\min} - D_{\max}/\eta\).

Now we first consider \(0 < X(t) \leq M_{UPS} - V_{\max}/T - M_{\min} - D_{\max}/\eta\), then \(R(t) = 0\). Since there is no battery recharging and the maximum discharged energy is \(D_{\max}/\eta\) each time, we have:

\[-V_{\max}/T - D_{\max}/\eta < -D_{\max}/\eta \leq X(t+1) \leq X(t) \leq M_{\upsilon} - V_{\max}/T - M_{\min} - D_{\max}/\eta.\]

Next, suppose \(-V_{\max}/T < X(t) \leq 0\), then \(D(\beta(t)) = 0\). The maximum charging and recharging energy each time are \(R_{\max} \eta\) and \(D_{\max}/\eta\), respectively. Thus we obtain:

\[-V_{\max}/T - D_{\max}/\eta < X(t+1) \leq X(t) + R_{\max} \eta - M_{\upsilon} - V_{\max}/T - M_{\min} - D_{\max}/\eta.\]

Finally, we consider the case of \(-V_{\max}/T - D_{\max}/\eta \leq X(t) \leq -V_{\max}/T\). Since \(X(t) \leq -V_{\max}/T\), \(D(\beta(t)) = 0\). Then \(-V_{\max}/T - D_{\max}/\eta \leq X(t) \leq X(t+1) \leq -V_{\max}/T + M_{\upsilon} - M_{\min} - D_{\max}/\eta\).

Then, from Eq. 9, we have:

\[-V_{\max}/T - D_{\max}/\eta \leq X(t) = m(t) - V_{\max}/T - M_{\min} - D_{\max}/\eta \leq M_{\upsilon} - V_{\max}/T - M_{\min} - D_{\max}/\eta.\]

It is easy to see that \(M_{\min} \leq m(t) \leq M_{\upsilon}\).

(3) Since MultiGreen makes decisions to satisfy all the constraints in problem \(P3\) and \(P4\), combining the constraints together, all the constraints of problem \(P1\) are satisfied. Therefore, MultiGreen control decisions are feasible to problem \(P1\). \(\square\)

**Remarks:** MultiGreen can approach the optimal solution of problem \(P1\) within a deviation of \(B_2/V\). As CSPs increase the value of \(V\), they can push the average cost to be arbitrarily close to the minimum value, according to a desired tradeoff between DPSS cost minimization and datacenter availability. The length of timeslot \(T\) decides how frequently MultiGreen performs energy procurement and battery charging and discharging. We will carry out detailed evaluations in Sec. 5.2.2 to show that even infrequent actions can still achieve significant cost reduction.

4.2 Robustness Analysis

Since MultiGreen approximates future queue backlog \(X(\tau)\) using its current level, an important question remained to be answered is: *is the performance still bounded if MultiGreen makes its decisions based on an approximated queue backlog \(\hat{X}(\tau)\) that is different from the actual value \(X(\tau)\)?* The dynamic UPS energy levels reflect the variation of energy demand and renewable energy supply. The following Theorem 3 demonstrates the robustness of MultiGreen in its performance to uncertainties of energy demand and supply.

**Theorem 3. (Robustness):** We assume that the estimated virtual battery level \(\hat{X}(\tau)\) and its actual level \(X(\tau)\) satisfy \(|\hat{X}(\tau) - X(\tau)| \leq \theta\). Then, if we use this approximated UPS battery level in the MultiGreen algo-
5.2 Analysis of Sensitivity on Critical Factors

We evaluate MultiGreen through trace-driven simulations with realistic parameters and one-month data on datacenter energy demand, renewable energy production and electricity prices.

5.1 Real-World Trace and Experimental Setup

Real-World Traces: To simulate the intermittent availability of renewable energy, we use solar energy data from the Measurement and Instrumentation Data Center (MIDC) [25]. Specifically, we use the meteorological data from Jan. 1st, 2012 to Jan. 31st, 2012 from central U.S.. To simulate the varying electricity prices, we use the electricity prices in central U.S. between Jan. 1st, 2012 and Jan. 31st, 2012, from the New York Independent System Operator (NYISO) [28]. Similar to [38], we use the energy demand from a Google Cluster including Web search and Webmail services. We regulate the data to our assumed datacenter by removing demand peaks above P_{grid}. The traces are shown in Fig. 3.

System Parameters: According to results in recent empirical studies, we assume that the limits of UPS charging and discharging rates are D_{max} = R_{max} = 0.5MW, and charging/discharging costs are C_r = C_d = 0.1 dollars [36]. The minimum battery level M_{min} is 5-minute worth of energy of the UPS [11]. The maximum number of UPS charge/discharge cycles is L_{UPS} = 5,000 with a 4-year lifetime constraint [11]. The efficiency of UPS charging/discharging is η = 0.8 [20]. We set the grid supply limit as P_{grid} = 2MW [36].

Algorithms for Comparison: We compare MultiGreen with the offline optimal algorithm (Optimal) and an online algorithm Green that solely leverages renewable energy production, without exploiting time-varying electricity prices. The Green algorithm also tries to maximize the usage of renewable energy, i.e., leveraging UPS battery to store excess renewable energy production for future need. However, the Green algorithm ignores the two-timescale grid markets, and does not store grid energy in UPS when the electricity prices are low and supply energy when the electricity prices are high.

5.2 Analysis of Sensitivity on Critical Factors

From Theorem 2, we note that the performance of
MultiGreen depends on parameters $V$ and $T$, battery capacity and the energy prices in the two-timescale grid markets. We conduct sensitivity analysis on these critical factors to characterize their impact on the DPSS operational cost.

5.2.1 Impact of Control Parameter $V$

As shown in Fig. 4, to simulate a 1-day-ahead power market, we fix $T$ to be 24 time slot and each fine-grained time slot is 1 hour, i.e., $N_T = 24$. We conduct experiments with different values of $V$, which show that as $V$ increases from 0.5 to 100, MultiGreen achieves a time-averaged cost that becomes closer to the optimal solution. This quantitatively confirms Theorem 2 that MultiGreen can approach the optimal solution within a diminishing gap of $O(1/V)$. In contrast, the Green algorithm has a constant cost that is irrelevant with $V$. Interestingly, the crossover between cost curves of MultiGreen and Green clearly captures the tradeoff between the average operational cost and constraint satisfaction. When $V < 7.48$, MultiGreen has a higher cost and a higher level of constraint satisfaction than Green. On the other hand, when $V > 7.48$, due to more frequent battery charging/discharging, MultiGreen has a lower cost and a lower level of constraint satisfaction than Green. By choosing an appropriate value of $V$, e.g., $V = 10$, MultiGreen can achieve a significantly lower cost compared with Green while guaranteeing acceptable satisfaction of constraints on datacenter availability and UPS lifetime.

5.2.2 Impact of Coarse-grained Time Frame $T$

In Fig. 5, we fix $V$ to be 10 and vary $T$ from 3 time slots (3 hours) to 144 time slots (6 days), which is a sufficient-long range for exploring the impact of different timescales of the grid’s long-term market. We observe that $T$ has relatively less impact on the cost of operating the DPSS. The fluctuation of the time-averaged cost is more notable when $T$ becomes longer. The rationale is that the term $B_T$ in Theorem 3 is proportional to $T$, which means that the uncertainties of energy demand and renewable energy increase with the increase of $T$. Nevertheless, the time-averaged cost only fluctuates within $[-9.7\%, +8.5\%]$. This corroborates Theorem 3 that, even with infrequent decisions of the DPSS operations, MultiGreen can still achieve significant cost reduction.

5.2.3 Impact of Battery Capacity and Grid Markets

In Fig. 6, we compare the time-averaged total cost under different battery sizes ($M_{UPS} \in \{0, 0.25, 0.5, 1\} \text{MWh}$) over the 31-day period with $V = 10$ and $T = 24$. It shows that the time-averaged total cost decreases with the increase of the UPS battery capacity. The rationale is that an UPS with larger capacity can store more surfluous renewable energy generated, or more energy purchased from the grid when the price is low, to serve the demand, resulting in lower overall costs.

In Fig. 6, we also compare the case with energy purchase in two-timescale markets with the case where only the real-time market exists, both with $V = 10, T = 24, M_{UPS} = 0.5 \text{MWh}$. We can observe that the existence of the grid’s long-term market can bring in additional cost reduction. The reason is that DPSS can purchase certain amount of energy beforehand in the grid’s long-term market with relative lower prices.

In addition, we can observe that even without the UPS battery, the MultiGreen algorithm with the two-timescale markets can reduce the cost by $10.06\%$, compared to the Green algorithm. With two markets, when we increase the battery size from 0 to 1MWh, the average operational cost reduction ranges from 10.06% to 34.21%. The benefit brought by energy storage is higher than that of exploiting the two markets. When the battery size is large enough, MultiGreen can approach the optimal offline algorithm.

5.3 Characterizing Algorithm Robustness

As mentioned in Sec. 3, our MultiGreen algorithm approximates the future queue statistics as the current values. Now we explore the influence of approximation errors on the performance of MultiGreen. We add a random approximation error to the datacenter energy
demand, solar energy generation and energy prices, e.g., uniformly distributed ±50% errors [39]. We let MultiGreen make all the control decisions based on the data set with such random errors under different values of $V$. In Fig. 8, we show the differences in percentage between the DPSS operational costs achieved with approximated values and the results we obtained using the original traces. We observe that the difference fluctuates within $[-1.3\%, 2.1\%]$ for all values of $V$. Thus, MultiGreen is robust to inaccurate future information.

Further, we study the impacts of renewable energy penetration (the percentage of renewable energy in the total datacenter energy supply) and the variation of datacenter energy demand on the total cost. In Fig. 7, x-axis represents renewable energy penetration in the range of $[0, 100\%]$. Y axis represents the standard deviation of the demand, i.e.,

$$\sqrt{\sum_{t=0}^{KT-1} [d(t) - \mathbb{E}(d)]^2 \times p_d(t)},$$

where $\mathbb{E}(d)$ is the expectation of the series of demand $d(t)$ over time length $t \in [0, KT]$, and $p_d(t)$ is the distribution probability of $d(t)$. We assume that the random variable of the datacenter energy demand is uniformly distributed ($p_d(t) = 1/KT$). As expected, Fig. 7 shows that with the increase of penetration of renewable energy, the DPSS operational cost decreases significantly. The rationale is that renewable energy is harvested cost-free (we do not consider the construction cost). In contrast, as the variation of demand increases, the operational cost increases slightly. The rationale is that intensive variation incurs large approximation errors.

6. RELATED WORK

In this section, we discuss the research most pertinent to this work as follows. The first category of works is exploiting renewable energy in datacenters. Many large IT companies recently consider greening their datacenters with renewable energy [7,8,10,22–24,35,41]. However, the intermittent nature of renewable energy poses significant challenges to make use of them. Some works make the traffic “follow the renewables” to execute workload when/where renewable energy is available [10, 22–24,41] or carbon footprint is low [7]. However, these approaches require prediction of renewable energy production when scheduling workload, or sacrifice performance to avoid wasting renewable energy. Other works supply renewable energy to deferrable loads to align demand with intermittent available renewable energy [27,29,30]. But they are from the prospective of renewable energy providers and do not consider energy storage and multiple markets in the smart grid.

The second category of works is leveraging energy storage in datacenters. Recently, UPS shows its benefits to reduce electricity costs in datacenters [11–13,36,38]. Datacenters can store energy in the UPS when energy prices are low and discharge UPS when prices are high, to reduce the power drawn from the grid [13,36]. Moreover, UPS can shave peaks [11,12]. During periods of low demand, UPS batteries store energy, while stored energy can be used to temporarily augment the grid supply during hours of peak load. However, these works focus on studying the benefits of UPS battery for power cost reduction, and no renewable energy and grid markets are considered. On the contrary, we leverage UPS to study how to manage multiple power supplies of a datacenter in an integrated way.

Third stream of works is on multiple timescale dispatch, pricing and scheduling in smart grid. Nair et al. [1] studied the optimal energy procurement from long-term, intermediate, and real-time markets under intermittent renewable energy supplies. Jiang et al. [19] proposed an optimal multi-period power procurement and demand response algorithm without energy storage. “Risk-limiting-dispatching” is proposed in [37] to manage integrated renewable energy. However, the above three approaches assume that the demand can be known ahead. Jiang et al. [21] solved the optimal day-ahead procurement and real-time demand response problem using dynamic programming, while He et al. [15] formulated the multi-timescale power dispatch and scheduling problem as a Markov decision problem. Both these approaches need substantial system statistics and are computationally expensive. We mitigate these disad-
vantages by applying two-stage Lyapunov optimization that makes online control decisions without \textit{a priori} knowledge or any stationary distribution of energy prices, demand and supply. Recently the authors in [13, 32, 33, 39, 41] distributed requests across multiple data centers to reduce electricity costs by leveraging both time diversity and location diversity of electricity prices in the smart grid. In contrast, we study how to reduce the operational cost in a datacenter powered by multiple power sources rather than how to distribute requests across datacenters.

In addition, interest has been growing in power management in smart grids and datacenters using Lyapunov optimization [5, 6, 9, 26, 42]. On smart grids, several works have proposed optimal power management based on single-stage Lyapunov optimization. However, they either focused on managing individual household demand [14] or did not consider the interaction between renewable energy and energy storage [14, 19, 20, 27]. In contrast, we manage the uncertain datacenter demand and multi-source energy supply in a systematic fashion using two-stage Lyapunov optimization. Although [34, 39] have used two-stage Lyapunov to design a two-timescale algorithm and a $T$-Step Lookahead algorithm, both of them study how to schedule jobs or distribute requests in solely grid-powered geographical datacenters rather than how to supply multi-source energy in a datacenter with uncertain demand.

7. CONCLUSION

In this paper, we study an important problem of how to minimize the operational cost of datacenters by using multiple energy resources. We propose MultiGreen, an online control algorithm applying the two-stage Lyapunov optimization technique, which optimally schedules multiple energy supply sources to power a datacenter, in a cost minimizing fashion. Without requiring \textit{a priori} knowledge of system statistics, MultiGreen can deliver reliable energy to datacenters while minimizing the operational cost in the datacenter’s long-run operation. Both mathematical analyses and trace-driven evaluations demonstrate the optimality and robustness of MultiGreen. Especially, it can approach the offline optimal cost within a diminishing gap of $O(1/V)$, which is mainly decided by the UPS battery capacity, grid market structure and DPSS operation frequency for energy purchasing and UPS charging/discharging.

8. REFERENCES


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