SmartDPSS: Cost-Minimizing Multi-source Power Supply for Datacenters with Arbitrary Demand

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Abstract—To tackle soaring power costs, significant carbon emission and unexpected power outage, Cloud Service Providers (CSPs) typically equip their Datacenters with a Power Supply System (DPSS) nurtured by multiple sources: (1) smart grid with time-varying electricity prices, (2) uninterrupted power supply (UPS), and (3) renewable energy with intermittent and uncertain supply. It remains a significant challenge how to operate among multiple power supply sources in a complementary manner, to deliver reliable energy to datacenter users with arbitrary demand over time, while minimizing a CSP’s operation cost over the long run. This paper proposes an efficient, online control algorithm for DPSS, SmartDPSS, based on the two-timescale Lyapunov optimization techniques. Without requiring a priori knowledge of system statistics, SmartDPSS allows CSPs to make online decisions on how much power demand, including delay-sensitive demand and delay-tolerant demand, to serve at each time, the amount of power to purchase from the long-term-ahead and real-time grid markets, and charging and discharging of UPS over time, in order to fully leverage the available renewable energy and time-varying prices from the grid markets, for minimum operational cost. We thoroughly analyze the performance of our online control algorithm with rigorous theoretical analysis. We also demonstrate its optimality in terms of operational cost, demand service delay, datacenter availability, system robustness and scalability, using extensive simulations based on one-month worth of traces from live power systems.

I. INTRODUCTION

Cloud service providers (CSPs) are typically facing three major problems with the operation of their datacenters: (1) Skyrocketing power consumption and electricity bills, e.g., Google (> 1,120GWh, > $67M) and Microsoft (> 600GWh, > $36M) [1]. (2) Serious environmental impact, as IT carbon footprints can occupy 2% of the global CO2 emissions reportedly [2]. (3) Unexpected power outages, e.g., Amazon experienced another outage in October 2012 affecting many sites due to failures in the power infrastructure [3].

To mitigate these issues, modern CSPs typically equip their datacenter power supply systems (DPSS) with multiple power sources in a complementary manner, as illustrated in Fig. 1. First, modern datacenters get the primary power from the smart grid which typically provides pricing schemes at different timescales, such as the long-term-ahead grid market and the real-time market [4]–[8]. Next, datacenters are equipped with uninterrupted power supply (UPS) to guarantee datacenter availability in terms of all-time operation with continuous power delivery. Finally, CSPs are also starting to green their datacenters by integrating on-site renewable energy, such as solar and wind energies [9]–[14]. The renewable energy is connected to the grid via a grid-tie device, which combines electricity produced from the renewable sources and the grid on the same circuit for power supply [13], [15]. The amount of renewable energy produced is varying significantly over time [2]. Since UPS battery is usually over provisioned to guarantee more than six 9’s datacenter availability [16], UPS can be used to stored energy during periods of high renewable generation and/or low electricity price from the grid markets. When the renewable power is insufficient or energy prices from the grid are high, the UPS battery can be discharged to serve the power demand [16]–[21].

When operating such a DPSS, several key control decisions need to be made in an online fashion, for long-term cost minimization of the cloud service provider: (1) Considering the power demand from datacenter users can be categorized into delay-sensitive demand and delay-tolerant demand, how much demand the DPSS should serve at each time, such that all delay-sensitive demand is served on time while delay-tolerant demand is served before the maximally allowed delay? (2) How much energy to be purchased from the long-term-ahead grid market and the real-time market, respectively? (3) How to use the UPS battery to store excess power generated/purchased and supply power when needed? It is challenging to optimally utilize the multiple sources to reliably power a datacenter, while minimizing its operational cost over the long term, in a dynamic system: on the demand side, power demand in a datacenter is arbitrary over time, given that workload arrivals may not follow any stationary distributions and diverse applications have variant resource needs; on the supply side, energy prices provided by the smart grid are time-varying as well, both for the long-term prices and the real-time prices, and the unpredictable nature of renewable energy adds onto the supply uncertainties.

There have been a number of works investigating data-
center power supply in cases of uncertain power demand, renewable energy supply and electricity prices from smart grids. They may either assume a priori knowledge of the power demand [5], [7], [8], [17], or require a substantial amount of statistics of the system dynamics, obtained based on excessive computational complexity [4], [6] or different forecast techniques [13], [14]. Some only investigate single-day or single-household power supply optimization [6], [22], while others may neglect the interaction among energy production/purchase, energy storage and demand management from the perspective of a datacenter operator [7], [17], [22]–[26]. To the contrast, we seek to design an efficient online control strategy for long-term optimal operation of the DPSS under arbitrary power demand and uncertain renewable energy supply in a synergetic manner, without requiring a priori knowledge or stationary distribution of system statistics.

Especially, we formulate a stochastic optimization model that minimizes the long-term operational cost of a datacenter under time-varying power demand and renewable energy production, two-timescale pricing schemes from the smart grid and a finite UPS battery capacity, and derive a practical and provably-efficient online DPSS control algorithm, SmartDPSS, based on the two-stage Lyapunov optimization technique [27]–[30]. The basic idea of the algorithm is to decide the amount of energy to purchase from the long-term-ahead grid market in intervals of longer periods of time, to tackle demand dynamics and energy price fluctuations in the future interval, and also to decide the amount of energy to purchase from the real-time grid market, as well as the amount of energy to store into or discharge from the UPS battery, in smaller time scales. For supply to the power demand, the algorithm addresses delay-sensitive demand when it is generated, and caters to the delay-tolerant demand in a more strategic fashion over time, while guaranteeing that it is served before the maximally allowed delay. The online decisions are set to best utilize the available renewable energy produced over time and the periods with lower electricity prices from the grid market, in order to minimize the operational cost in the long run of the datacenter, which is the sum of costs for grid power purchase, UPS operation, renewable energy production and energy waste.

Not requiring any a priori knowledge of the system dynamics, SmartDPSS can arbitrarily approach the optimal offline cost, computed with full knowledge of the system over its long run. Specifically, SmartDPSS can obtain a time-average cost within a deviation of $O(1/V)$ from the optimum, while guaranteeing datacenter all-time availability and bounding the demand service delay by $O(V)$, where $V$ is a logarithmic parameter deciding the cost-delay tradeoff. We thoroughly analyze the performance of our online control algorithm with rigorous theoretical analysis. We also demonstrate its optimality (in terms of a well balanced tradeoff among DPSS operation cost, demand service delay, and the UPS lifetime), system stability (in terms of robustness and adaptivity to time-varying power demand and supply) and scalability (in terms of adaptivity to the ever-increasing power demand and renewable energy supply), using extensive simulations based on one-month worth of traces from live power systems.

## II. System Model and Control Objective

Without loss of generality, we consider a DPSS system that operates in a discrete-time mode. Time is divided into $K (K \in \mathbb{N}^+)$ coarse-grained time slots of length $T$ each, in accordance with the length of the long-term-ahead grid market, e.g., days or hours [6]. Each coarse-grained time slot is further divided into $T (T \in \mathbb{N}^+)$ fine-grained time slots, e.g., $T = 5$ in Fig. 2. Empirically, each fine-grained time slot is 15 or 60 minutes long, for the datacenter to adjust its power control strategies in a more prompt fashion [7], [29].

### A. Online Control Decisions

1) Two-timescale control decisions on the supply side: As illustrated in Fig. 2, at the beginning (first fine-grained time slot) of each coarse-grained time slot $t = kT$ ($k = 1, 2, ..., K$), DPSS observes the demand $d(t)$ and renewable power $r(t)$ during time slot $t$. Then DPSS makes a decision of how much additional energy $g_{be}(t)$ to be purchased at present from the real-time grid market at a price $p_{lt}(t)$ (with an upper bound price $P_{max}$) in the long-term-ahead market. Thus DPSS schedules energy $g_{be}(t)/T$ for each fine-grained time slot in the next coarse-grained time slot. At each fine-grained time slot $\tau \in [t, t + T - 1]$, according to the actual generated renewable energy $r(\tau)$ and demand $d(\tau)$ during time slot $t$, DPSS decides how much amount of additional energy $g_{rt}(\tau)$ to be purchased from the real-time grid market at a price $p_{rt}(\tau)$ (0 $\leq p_{rt}(\tau) \leq P_{max}$). Thus, we get:

$$s(\tau) = g_{be}(t)/T + g_{rt}(\tau) + \gamma(\tau), 0 \leq s(\tau) \leq S_{max}. \quad (1)$$

2) Control decisions on the demand side: As shown in Fig. 3, datacenter power demand consists of two arbitrary and independent parts, i.e., $d(\tau) = d_{ds}(\tau) + d_{dt}(\tau)$: delay-sensitive power demand $d_{ds}(\tau)$ that is served on time and delay-tolerant demand $d_{dt}(\tau)$ ($0 \leq d_{dt}(\tau) \leq D_{max}$) that is served before the maximally allowed delay $\gamma_{max}$. As illustrated in Fig. 4, delay-tolerant demand is stored in a queue $Q(\tau)$ with the arrival rate $d_{at}(\tau)$ and service rate $\gamma(\tau) Q(\tau)$. $\gamma(\tau) (\gamma(\tau) \in [0, 1])$ is the decision made by DPSS, representing how much demand backlog in $Q(t)$ to serve at each fine-grained time slot $\tau$. The rest demand is deferred to later times with more available renewable energy or lower energy price. Let $s_{dt}(\tau) = \gamma(\tau) Q(\tau) (0 \leq s_{dt}(\tau) \leq S_{max})$ denote the energy supply for delay-tolerant demand at each fine-grained time slot $\tau$, then we have:

$$Q(\tau + 1) = \max\{Q(\tau) - s_{dt}(\tau), 0\} + d_{dt}(\tau). \quad (2)$$

![Fig. 2. An example of two-timescale energy supply. Each coarse-grained time slot is divided into 5 fine-grained time slots. T controls DPSS operation frequency. When T = 1, our model is reduced to single time scale.](image-url)
3) Queue dynamics of battery level: Correspondingly, the supply and demand side control decisions decide the operations of the battery: whether to recharge battery \( b_r(\tau) \) to store energy or discharge battery \( b_d(\tau) \) to meet demand. If the energy supply is greater than served demand, then the surplus energy is stored in the battery, i.e., \( b_r(\tau) = [s(\tau) - d_{ds}(\tau) - \gamma(\tau)Q(\tau))]^+ \). If the power demand is greater than the supply, then the battery is discharged to supplement the supply, i.e., \( b_d(\tau) = [d_{ds}(\tau) + \gamma(\tau)Q(\tau) - s(\tau)]^+ \). \( \eta_c(\eta_c \in [0, 1]) \) is the recharging efficiency and \( \eta_d(\eta_d \geq 1) \) is discharging efficiency. Let \( B_{max} \) be the maximum battery capacity. The battery level process \( b(\tau) \) can be concisely defined as:

\[
b(\tau + 1) = \min[B_{max}, b(\tau) + b_r(\tau)\eta_c - b_d(\tau)\eta_d]. \tag{3}
\]

In addition, battery energy plus the supply may exceed the power demand due to either high renewable production and/or low demand. The superfluous energy is wasted if it can not be stored in battery due to limited size. We define the waste of energy \( W(\tau) \) as: \( W(\tau) = [b(\tau) + s(\tau) - d(\tau) - B_{max}]^+ \).

B. Constraints

In our model, we consider power demand \( d(\tau) \), renewable supply \( r(\tau) \) and electricity prices \( p_{bef}(t), p_{rt}(t) \) are random variables without any probabilistic assumptions. There are a series of constraints in the above decision-making.

1) Matching demand and supply: At any fine-grained time slot \( \tau \), the aggregated power supply should be equal to the served power demand:

\[
s(\tau) + b_{ds}(\tau) - b_{rc}(\tau) = d_{ds}(\tau) + \gamma(\tau)Q(\tau). \tag{4}
\]

2) Balancing procurement accuracy and cost: Intuitively, the closer to real-time, the DPSS can make more accurate decision on power purchasing for time-varying demand. However, in practice, the price of electricity in real-time market tends to be higher on average than that in long-term-ahead market, i.e., \( p_{rt}(t) > p_{rt}(t) \) [5], [6], [8]. The rationale is that upfront payment is associated with cheaper contract prices in the long-term market. Hence, when procuring power in two-timescale markets, the DPSS should make the best tradeoff between procurement accuracy and power cost. Additionally, we assume that the maximal amount of power that the datacenter can draw from grid at each time is limited by \( P_{grid} \):

\[
0 \leq g_{bef}(t)/T + g_{rt}(t) \leq P_{grid}. \tag{5}
\]

3) Guaranteeing delay-tolerant demand deadline: Since queuing delay is closely related to the queue backlog, we bound the length of demand queue \( Q(t) \), which in turn determines the delay performance. We use \( Q = \lim_{\tau \to \infty} \frac{1}{\tau} \sum_{\tau=0}^{\tau-1} [Q(\tau)] \) to denote the time-average expected backlog of delay-tolerant demand. To guarantee the maximal deadline \( \lambda_{max} \), any control policy should satisfy:

\[
\forall \tau, Q(\tau) < Q_{max}, Q < \infty, \tag{6}
\]

where \( Q_{max} \) is the maximal backlog. We will specify that \( \lambda_{max} \) is a proportional function of \( Q_{max} \) in Lemma 2 in Sec. III-C. Therefore, when queue \( Q(t) \) backlog is bounded, the maximal deadline is satisfied.

4) Ensuring datacenter availability: To avoid discretionary UPS discharging, we assume that under any feasible control algorithm, UPS battery has a minimum energy level \( B_{min} \) to offer reliable datacenter operation in case of power outage. Let \( B_{min} \leq b(\tau) \leq B_{max} \). \( \tag{7} \)

Empirically, \( B_{min} \) can power the peak demand of a datacenter for about a minute, while \( B_{max} \) can vary within 5 ~ 30 minutes when powering peak datacenter demand [16].

5) Considering UPS lifetime and operation cost: In practice, UPS battery has constraints on the maximum amounts of power by which we can recharge or discharge per time:

\[
0 \leq b_{rc}(\tau) \leq B_{max}^c, 0 \leq b_{dc}(\tau) \leq B_{max}^d. \tag{8}
\]

where \( B_{max}^c \) and \( B_{max}^d \) are the maximum amounts of UPS energy recharging and discharging, respectively. Each time, battery is either charged or discharged: \( b_{rc}(\tau) \cdot b_{dc}(\tau) = 0 \).

It has been practically shown that UPS lifetime is a decreasing function of charge/discharge cycles [18]. To avoid aggressive charging/discharging, we assume that the maximum allowable discharging/charging number \( N_{max} \) during the horizon \( t \in KT \) satisfies:

\[
0 \leq \sum_{\tau=0}^{t-1} n(\tau) \leq N_{max}, \tag{9}
\]

where \( n(\tau) = 1 \) if \( b_{rc}(\tau) > 0 \) or \( b_{dc}(\tau) > 0, 0 \) otherwise.

The cost of repeated operation of the battery is a function of how often/much it is charged and discharged. We assume that the costs of battery charging and discharging per time are the same, denoted as \( C_b \). If a UPS costs \( C_{buy} \) to purchase and it can sustain \( C_{cycle} \) charge/discharge cycles, then \( C_b = C_{buy}/C_{cycle} \). At time \( \tau \), the UPS operation cost is \( n(\tau)C_b \).

C. Stochastic Constrained Cost Minimization Problem

At each fine-grained time slot \( \tau \), DPSS operational cost is the sum of costs for grid power purchasing, UPS charging/discharging, renewable energy producing and wasted energy. However, the primary costs for solar panels and wind turbines are construction costs, and their operation cost is negligible [10]. Therefore, \( Cost(\tau) \triangleq g_{bef}(t)/T p_{uf}(t) + g_{rt}(\tau)p_{rt}(\tau) + n(\tau)C_b + W(\tau) \). Our objective is to design a flexible and robust online DPSS control policy that automatically adapts to the time-varying system for solving the following stochastic cost minimization problem \( \text{P1} \):

\[
\min_{g_{bef}, g_{rt}, \gamma} \left\{ \text{Cost}_{av} \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[Cost(\tau)] \right\}, \tag{10}
\]

s.t. constraints (4)/(5)/(6)/(7)/(8)/(9).
Since the battery can be charged to store energy or discharged to serve demand, the current control decisions are coupled with the future decisions. For example, current decisions may delay excessive demand and hence block future demand service, or leave insufficient battery capacity for storing future renewable energy, or overuse battery and threaten datacenter availability. To solve this long-term optimization problem, the commonly used dynamic programming technique [31] suffers from a curse of dimensionality, and requires significant knowledge of the demand and supply probabilities. In contrast, the recently developed Lyapunov framework [27], [28] is shown to enable the design of online control algorithms for such constrained optimization of time-varying systems without requiring a priori knowledge of the workload and cost statistics. In particular, our above model of the two-timescale power delivery structure well fits the two-stage Lyapunov optimization framework [29], that can enable us to perform two levels of control strategies for two levels of granularity. Therefore, we design our online control algorithm based on two-timescale Lyapunov optimization.

D. An Optimal Offline Algorithm

Here we first present an optimal offline solution for problem \( \text{P1} \) as a benchmark for comparison. In the theoretically optimal scenario, DPSS knows all future system statistics including a benchmark for comparison. In the theoretically optimal two-timescale Lyapunov optimization.

Therefore, we design our online control algorithm based on the following drift-plus-penalty term every \( t \)-slot conditional Lyapunov function

\[
L(\Theta(t)) \triangleq \frac{1}{2} [Q^2(t) + X^2(t) + Y^2(t)].
\]

Then, the \( T \)-slot conditional Lyapunov drift is defined as:

\[
\Delta_T (\Theta(t)) \triangleq \mathbb{E}[L(\Theta(t + T)) - L(\Theta(t))|\Theta(t)].
\]

Following the Lyapunov framework of drift-plus-penalty algorithm [27], our control algorithm is designed to make decisions on \( g_{bf}(t), g_r(t), \lambda(t) \) to minimize an upper bound on the following drift-plus-penalty term every \( T \) slots:

\[
\Delta_T (\Theta(t)) + V \mathbb{E}\{\sum_{\tau=t}^{t+T-1} \text{Cost}(\tau)|\Theta(t)\},
\]

where the control parameter \( V \) is chosen according to the CSP to give different weights to DPSS operation cost and demand service delay. A large delay can reduce operational cost.

A. Workload Delay-Aware Virtual Queue

In order to guarantee the maximum delay \( \lambda_{\text{max}} \) for delay-tolerant demand, we define a delay-aware virtual queue \( Y(t) \) based on the technique of \( \epsilon \)-persistent queue [23] as below:

\[
Y(t+1) = \max\{Y(t) - s_d(t) + \epsilon_1 Q(t), 0\},
\]

where \( Q(t) \) is an indicator variable that is 1 if \( Q(t) > 0 \) and 0 otherwise. \( \epsilon \) is a positive parameter that ensures that \( Y(t) \) grows whenever there is power demand in \( Q(t) \) that has not been serviced. Under any feasible algorithm, we should ensure that the deferred power demand can be served within a worst case delay \( \lambda_{\text{max}} \) given in Lemma 2.

**Lemma 2**: For any time slot \( t \), if the system can be controlled to ensure that \( Q(t) < Q_{\text{max}} \) and \( Y(t) < Y_{\text{max}} \), then any delay-tolerant demand is fulfilled with a maximum delay \( \lambda_{\text{max}} \) defined as follows:

\[
\lambda_{\text{max}} \triangleq \left\lceil \frac{(Q_{\text{max}} + Y_{\text{max}})}{\epsilon} \right\rceil.
\]

B. Datacenter Availability-Aware Virtual Queue

To guarantee datacenter availability and deliver reliable energy to datacenter power demand, we should guarantee constraint (7) of battery level. We use an auxiliary variable \( X(t) \) to track the battery level, defined as follows:

\[
X(t) = b(t) - U_{\text{max}} - B_{\text{min}} - B_{\text{max}}^d,
\]

where \( U_{\text{max}} \) is the upper bound of the sum \( Q(t) + Y(t) \) as described later in Sec. V-A. The intuition behind \( X(t) \) is that by carefully tuning the maximum queue backlog \( U_{\text{max}} \) for decision-making, we can push the battery level to values above the lower bound \( (B_{\text{min}}) \) to avoid blackout. Recall that \( b(t) \) is the actual battery level in time slot \( t \) and evolves according to Eq. (3). The dynamics of \( X(t) \) is given as:

\[
X(t+1) = \min\{B_{\text{max}}, X(t) + b_{rc}(t)\eta_c - b_{dc}(t)\eta_d\}.
\]

In Theorem 2, we will prove that for any time slot \( t \), the queue \( X(t) \) is deterministically lower and upper bounded, and battery level \( b(t) \) is always in the safe range \([B_{\text{min}}, B_{\text{max}}]\).

C. Two-Timescale Lyapunov Optimization

Let \( \Theta(t) = [Q(t), X(t), Y(t)] \) be a concatenated vector of the actual and virtual queues. We define a quadratic Lyapunov function as:

\[
L(\Theta(t)) \triangleq \frac{1}{2} [Q^2(t) + X^2(t) + Y^2(t)].
\]

Then, the \( T \)-slot conditional Lyapunov drift is defined as:

\[
\Delta_T (\Theta(t)) \triangleq \mathbb{E}[L(\Theta(t + T)) - L(\Theta(t))|\Theta(t)].
\]

Following the Lyapunov framework of **drift-plus-penalty** algorithm [27], our control algorithm is designed to make decisions on \( g_{bf}(t), g_r(t), \lambda(t) \) to minimize an upper bound on the following drift-plus-penalty term every \( T \) slots:

\[
\Delta_T (\Theta(t)) + V \mathbb{E}\{\sum_{\tau=t}^{t+T-1} \text{Cost}(\tau)|\Theta(t)\},
\]

where the control parameter \( V \) is chosen according to the CSP to give different weights to DPSS operation cost and demand service delay. A large delay can reduce operational cost.
cost, yet can incur adverse effects on delay performance and datacenter availability. The following Theorem 1 gives the analytical basis on the drift-plus-penalty term.

**Theorem 1:** (Drift-plus-Penalty Bound) Let $V > 0$, $\epsilon > 0$, $T \geq 1$ and $t = kT, \tau \in [t, t + T - 1]$. To ensure two-timescale power purchasing $0 \leq g_{be,f}(t)/T + g_{rl}(\tau) \leq P_{grid}$, demand management decision $\gamma(\tau) \in [0, 1]$, battery level $b(\tau) \in [B_{min}, B_{max}]$ and demand backlog $Q(t) < Q_{max}$, under any operation actions, the drift-plus-penalty satisfies:

$$\Delta T(\Theta(t)) + V E\{\sum_{\tau=t}^{t+T-1} Cost(\tau)|\Theta(t)\} \leq H_1 T + V E\{\sum_{\tau=t}^{t+T-1} Cost(\tau)|\Theta(t)\}$$

$$- E\{\sum_{\tau=t}^{t+T-1} Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)]|\Theta(t)\}$$

$$+ E\{\sum_{\tau=t}^{t+T-1} X(\tau)[b_{rc}(\tau) - b_{dc}(\tau)]|\Theta(t)\}$$

$$+ E\{\sum_{\tau=t}^{t+T-1} Y(\tau)[e - s_{dt}(\tau)]|\Theta(t)\}$$

where $H_1 = S_{dt}^{max} + \frac{1}{2} [D_{max}^{dc} + B_{max}^{dc} + B_{max}^{rc} + \epsilon^2]$. We can see that the queue $\Theta(t)$ is deterministically upper bounded, implying that the constraints of worst case delay and datacenter availability will be guaranteed.

IV. SMARTDPSS: ONLINE CONTROL ALGORITHM DESIGN

Now we develop an online algorithm which attempts to achieve near-optimal solution without future statistics.

A. Relaxed Optimization Problem

The key principle of Lyapunov optimization framework is to choose control policies to minimize the right-hand-side (RHS) of (19). However, the CSP needs to know the future concatenated queue backlog $\Theta(t) = [Q(t), X(t), Y(t)]$ over time frame $\tau \in [t, t + T - 1]$. $\Theta(t)$ depends on UPS energy level $b(t)$, the power demand $d(t)$ and available renewable energy $r(t)$. The highly variable nature of power demand, renewable energy and electricity prices has been a major obstacle to make accurate decisions. In practice, system operators can use forecast techniques to predict the future statistics. However, an 90th percentile 1-hour ahead forecast error of renewable energy can be 22.2% [13].

Therefore, due to the continuous variation of the system, we instead approximate near future statistics as the current statistics, so as to avoid the computational complexity and substantial optimization requirement of forecasts, i.e., $Q(\tau) = Q(t), X(\tau) = X(t)$ and $Y(\tau) = Y(t)$ for $t < \tau \leq t + T - 1$. This greatly reduces the computational complexity of our algorithm. However, the simplification forces a "loosening" of the upper bound on the drift-plus-penalty, as proved in Corollary 1. We will prove that our algorithm is robust against the approximation in Sec. V-B.

**Corollary 1:** (Loosening Drift-plus-Penalty Bound) Let $V > 0$, $\epsilon > 0$ and $T \geq 1$. Considering Theorem 1 under approximation, the drift-plus-penalty term satisfies:

$$\Delta T(\Theta(t)) + V E\{\sum_{\tau=t}^{t+T-1} Cost(\tau)|\Theta(t)\} \leq H_2 T + V E\{\sum_{\tau=t}^{t+T-1} Cost(\tau)|\Theta(t)\}$$

$$- E\{\sum_{\tau=t}^{t+T-1} Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)]|\Theta(t)\}$$

$$+ E\{\sum_{\tau=t}^{t+T-1} X(\tau)[b_{rc}(\tau) - b_{dc}(\tau)]|\Theta(t)\}$$

$$+ E\{\sum_{\tau=t}^{t+T-1} Y(\tau)[e - s_{dt}(\tau)]|\Theta(t)\}$$

where $H_2 = H_1 + T(T - 1)B_{max}^{rc} + 2(T(T - 1)\epsilon^2$. Substituting the definitions of $Cost(\tau)$ and $s_{dt}(\tau)$ into the RHS of Eq. (20), we have the following relaxed problem P3:

$$\min_{g_{be,f}, g_{rl,\gamma}} E\{\sum_{t=1}^{t+T-1} \frac{g_{be,f}(t)}{T}[V_{pt}(t) - Q(t) - Y(t)]|\Theta(t)\}$$

$$+ E\{\sum_{t=1}^{t+T-1} g_{rl}(t)[V_{pr}(t) - Q(t) - Y(t)]|\Theta(t)\}$$

$$+ E\{\sum_{t=1}^{t+T-1} \gamma(\tau)[Q(t)^2 - Q(t)Y(t)]|\Theta(t)\}$$

$$+ E\{\sum_{t=1}^{t+T-1} [Q(t) + X(t) + Y(t)][b_{rc}(\tau) - b_{dc}(\tau)]$$

$$+ V n(\tau)C_b + VW(\tau)|\Theta(t)\}$$

s.t.

(4)(5)(6)(7)(8)(9).

B. Two-Timescale Online Control Algorithm

Our online two-timescale DPSS control algorithm SmartDPSS is illustrated in Algorithm 1. SmartDPSS observes current system statistics $[Q(t), X(t), Y(t), b(t)]$, and chooses $[g_{be,f}(t), g_{rl}(\tau), \gamma(\tau)]$ to minimize P3. Specifically, at the beginning of each coarse-grained time slot $t = kT$, SmartDPSS decides how much amount of energy $g_{be,f}(t)$ to be purchased from the long-term-ahead market based on delay-sensitive demand $d_{ds}(t)$ and renewable energy $r(t)$. At each fine-grained time slot $\tau \in [t, t + T - 1]$, SmartDPSS performs real-time procurement $g_{rl}(\tau)$, delay-tolerant demand management $\gamma(\tau)$ and resulted battery discharging $b_{dc}(\tau)$ and charging $b_{rc}(\tau)$ to balance demand and supply. At the end of each fine-grained time slot, SmartDPSS updates its queue statistics.

**Remark:** SmartDPSS is computationally efficient: each time it only has to solve linear programs with three variables $[g_{be,f}(t), g_{rl}(\tau), \gamma(\tau)]$ with six linear constraints (4)(5)(6)(7)(8)(9), without the need of a priori knowledge about processes of demand, renewable production and prices. We can easily solve the two sub-problems P4 and P5 using classical linear programming approaches, e.g., simplex method [33]. SmartDPSS enables CSPs to have a tunable system with the flexibility to make tradeoff between DPSS operation cost and demand service delay while meeting requirements of UPS lifetime and datacenter availability by appropriately tuning the cost-delay parameter $V$, delay control parameter $\epsilon$ and operation frequency $T$. 
Algorithm 1: SmartDPSS Online Control Algorithm.

1) Long-term Ahead Planning: At each time $t = kT (k \in \mathbb{Z}_+)$, observing system states $Q(t), Y(t)$, renewable energy $r(t)$, power demand $d_{ds}(t)$, maximum available battery level $b(t)$ and energy prices $p_{lt}(t)$, DPSS decides the optimal power procurement in the long-term market $g_{bef}(t)$ to minimize the following problem $P4$:

$$\min_{g_{bef}} \quad g_{bef}(t) \left[ V_{p_{lt}}(t) - Q(t) - Y(t) \right]$$

s.t. $g_{bef}(t)/T + r(t) + b(t) \geq d_{ds}(t)$,

$$0 \leq g_{bef}(t)/T \leq P_{grid}, B_{min} \leq b(t) \leq B_{max}.$$

2) Real-time Balancing: At each fine-grained time slot $\tau \in [t, t + T - 1]$, with system statistics $Q(t), X(t), Y(t)$, renewable production $r(\tau)$, power demand $d(\tau)$ and energy prices $p_{rt}(\tau)$, DPSS performs real-time procurement $g_{rt}(\tau)$ and delay-tolerant demand management decision $\gamma(\tau)$ to minimize the following optimization problem $P5$:

$$\min_{g_{rt}, \gamma} \quad \sum_{\tau=t}^{t+T-1} \left\{ g_{rt}(\tau) \left[ V_{p_{rt}}(\tau) - Q(\tau) - Y(\tau) \right] + \gamma(\tau) \left[ Q(\tau) - X(\tau) + Y(\tau) \right] \right\}$$

s.t. $(4)(5)(6)(7)(8)(9)$.

3) Queue Update: Update the actual and virtual queues using Eq. (2) (3) (12) (15).

C. Algorithm Properties and Overhead

SmartDPSS works at two different timescales, which is important to hedging against uncertainties of renewable production and energy prices [4], [6], [24]. We first observe that subproblem $P5$ has the following properties related to battery operation that is useful for later performance analysis:

**Lemma 3:** If $X(t) > 0$, then $b_{rc}(t) = 0$; if $X(t) < -[Q(t) + Y(t)]_{max} = -U_{max}$, then $b_{dc}(t) = 0$.

We highlight four other SmartDPSS’s interesting properties:

1) SmartDPSS is not work-conserving. Due to a high renewable energy price or low renewable energy production, the DPSS may choose not to serve certain delay-tolerant demand in particular time slot, even if $Q(t) > 0$. This will introduce additional delay but can reduce DPSS operational cost. (2) SmartDPSS can improve the return on investment of renewable source construction and UPS equipment by efficiently utilizing renewable energy and always existed UPS. (3) SmartDPSS can provide opportunities to reduce datacenter operational cost of cooling system due to lower power consumption. (4) However, SmartDPSS may incur power peaks due to its goal of executing as much demand as possible during periods of more available renewable energy and lower electricity price. Since we have set constrain of the maximum amount of power that the datacenter can draw from grid by $P_{grid}$. SmartDPSS has limited power peak emergencies. Incorporating cooling cost and power peaks management are part of our future work.

V. SMARTDPSS: PERFORMANCE ANALYSIS

In this section, we analyze the performance bound, robustness and scalability of our SmartDPSS algorithm.

To analyze the performance bound, we first have to characterize the optimal time-average cost $\phi^{opt}$ that can be achieved by our algorithm. Lemma 4 present a general optimal stationary randomized algorithm $\pi$ with the following structure:

**Lemma 4:** (Characterizing Optimal Stationary, Randomized Policy): If the datacenter power demand $d_{ds}(\tau), d_{dt}(\tau)$, renewable energy $r(\tau)$ and energy prices $p_{lt}(\tau), p_{rt}(\tau)$ are i.i.d. over slots, then there exists a stationary randomized control policy $\pi$ that takes control decisions $[g_{bef}(\tau), g_{rt}(\tau), \gamma(\tau)]$ while satisfying constraints $(4)(5)(6)(7)(8)(9)$, achieving the following guarantees:

$$\mathbb{E} \{ \text{Cost}_{\pi} \} = \phi^{opt}, \mathbb{E} \{ b_{rc}(\tau) \} = 0, \mathbb{E} \{ b_{dc}(\tau) \}.$$

A. SmartDPSS Performance Bound

We now analyze the performance of SmartDPSS in terms of the gap between the result achieved by SmartDPSS and the theoretical optimal solution $\phi^{opt}$ of cost minimization problem $P1$. We define an upper bound $V_{max}$ for the value of $V$ that can take in our algorithm:

$$V_{max} \triangleq T(B_{max} - B_{min} - B_{dt}^{max} - D_{dt}^{max} - \epsilon)/P_{max}.$$

Then, the following Theorem 2 gives the analytical bound.

**Theorem 2:** (Performance Bound Analysis): Given any fixed control parameter $0 < V \leq V_{max}, \epsilon > 0$ and long-term time slot $T \geq 1$ for $t \in KT$, SmartDPSS achieves:

1) The queue $X(t)$ for datacenter availability is deterministically lower and upper bounded as:

$$X(t) \geq -U_{max} - B_{dt}^{max}, \quad (21)$$

$$X(t) \leq B_{max} - U_{max} - B_{min} - B_{dt}^{max}. \quad (22)$$

2) The actual UPS battery level $b(t)$ is always in the range $[B_{min}, B_{max}]$. Hence, datacenter availability is satisfied.

3) The deferred delay-tolerant demand queue $Q(t)$ and delay-aware queue $Y(t)$ is deterministically bounded as:

$$Q_{max} \triangleq VP_{max}/T + D_{dt}^{max}, \quad (23)$$

$$Y_{max} \triangleq VP_{max}/T + \epsilon. \quad (24)$$
Further, the sum $Q(t) + Y(t)$ is also bounded by $U_{\text{max}}$:

$$U_{\text{max}} \triangleq VP_{\text{max}}/T + D_{\text{max}}^{dt} + \epsilon. \quad (25)$$

(4) The worst case delay for delay-tolerant demand is:

$$\lambda_{\text{max}} \triangleq \left[ (2VP_{\text{max}}/T + D_{\text{max}}^{dt} + \epsilon) / \epsilon \right]. \quad (26)$$

(5) The time-average cost $\text{Cost}_{av}$ achieved by SmartDPSS algorithm satisfies the following bound:

$$\text{Cost}_{av} \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[\text{Cost}(\tau)] \leq \phi^{opt} + \frac{H_2}{V}, \quad (27)$$

where $H_2$ is given in Corollary 1.

(6) All control decisions of SmartDPSS are feasible.

**Remark:** Theorem 2 demonstrates the $[O(1/V), O(V)]$ cost-delay tradeoff, where the time-average cost is within a deviation of $H_2/V$ of the optimal time-average cost while the worst case delay is bounded by $V/(T\epsilon)$. CSPs can push the time-average cost arbitrarily close to the optimal value by increasing the control parameter $V$. However, this increases the maximum queue backlog and worst case delay linearly in $V$. In addition, smaller $\epsilon$ implies larger delay and lower cost. The time slot $T$ decides how frequently SmartDPSS performs actions of power management. Smaller $T$ can obtain smaller time-average cost but incur larger queue backlog and service delay. This quantitative relations enable SmartDPSS to make flexible design choices and find the sweet point of the values of parameters $V, \epsilon$, and $T$.  

**B. SmartDPSS Algorithm Robustness**

Since SmartDPSS approximates future concatenated queue backlog $\Theta(t) = [Q(t), X(t), Y(t)]$ as its current level, it is robust for SmartDPSS to make its decisions based on the approximated queue backlog $\hat{\Theta}(t)$ that is different from the actual value $\Theta(t)$? The following Theorem 3 demonstrates the robustness of SmartDPSS to uncertainties of datacenter power demand and supply.

**Theorem 3:** (SmartDPSS Robustness): We assume that the approximated queue backlog $\hat{Q}(t), \hat{X}(t), \hat{Y}(t)$ and their actual value $Q(t), X(t), Y(t)$ satisfy $\hat{Q}(t) - Q(t) \leq \theta_{\text{max}}, |\hat{X}(t) - X(t)| \leq \theta_{\text{max}}$ and $|\hat{Y}(t) - Y(t)| \leq \theta_{\text{max}}$. Then, under SmartDPSS algorithm, the queues’ bound is unchanged. For the time-average operation cost, we can obtain:

$$\text{Cost}_{av}^{\text{Robust}} \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[\text{Cost}(\tau)] \leq \phi^{opt} + \frac{H_3}{V}, \quad (28)$$

where $H_3 = H_2 + T\theta_{\text{max}}$, and $H_2$ is given in Corollary 1.

**Remark:** By comparing the inequalities (27) and (28), we know that we need to set $V$ to a larger value to obtain the same operation cost compared with that has more accurate statistics. However, this will result in larger queue backlog and service delay. Hence, SmartDPSS works even with inaccurate information, but the robustness is achieved at the expense of a tradeoff between minimization of operation cost and constraints satisfaction of worst delay and datacenter availability.

**C. SmartDPSS Algorithm Scalability**

With the prosperous Internet services, CSPs have an ever-increasing datacenter power demand. Meanwhile, renewable energy will enjoy a boom in the coming years [2]. Thus, we consider a system expanding scenario that power demand and renewable energy expand to $\beta$ ($\beta \geq 1$) times of current scale. We assume that the uncertainty of the concatenated queue backlog $\Theta(t) = [Q(t), X(t), Y(t)]$ expands $\beta^\alpha$ ($\alpha \in [1/2, 1]$) [5]. We assume that UPS cannot be enlarged proportionally and stays fixed due to limits of space and capital cost. We have the following expanding model:

$$d(\beta, t) = \beta d(t), r(\beta, t) = \beta r(t), |\hat{\Theta}(t) - \Theta(t)| \leq \beta^\alpha \theta_{\text{max}},$$

where $\alpha$ denotes the similarity of workload demand and correlation of renewable sources. For instance, $\alpha = 1$ corresponds to an expanding scenario that the renewable energy sources are co-located and workloads are the same, and hence the uncertainties expand proportionately with $\beta$. The impact of system expanding is given by Corollary 2.

**Corollary 2:** (SmartDPSS Scalability) Under the system expanding model, the parameters $H_1(\beta) = \beta H_1$ in Theorem 1, $H_2(\beta) = \beta H_2$ in Theorem 2, and $H_3(\beta) = \beta H_3 + T\beta^\alpha \theta_{\text{max}}$ in Theorem 3.

**Remark:** Corollary 2 reveals that performance parameters increase linearly with system expanding. Thus, SmartDPSS adapts well with system expanding. But the expanding also impacts the robustness proportionately. Hence, CSPs need larger battery and more accurate power procurement to balance expanding demand and supply to improve system stability and cost-efficiency. Further, CSPs should offer diverse services rather than similar services to alleviate demand uncertainty, and leverage geo-distributed independent renewable energies rather than single source to alleviate supply uncertainty.

**VI. SMARTDPSS: PERFORMANCE EVALUATION**

We evaluate the performance of SmartDPSS through trace-driven simulation with realistic parameters and 1-month data on power demand, solar energy and electricity prices.

**A. Real-World Traces and Experimental Setup**

**Real-World Traces:** First, to simulate the intermittent availability of renewable energy, we use solar energy data from the Measurement and Instrumentation Data Center (MIDC) [34]. Specifically, we use the meteorological data from Jan. 1st, 2012 to Jan. 31st, 2012 from central U.S.. Second, to simulate the variable electricity prices, we use the central U.S. price traces also from Jan. 1st, 2012 to Jan. 31st, 2012 from the
New York Independent System Operator (NYISO) [35]. Third, similar to [19], we use the power demand from a Google Cluster including delay-sensitive Websearch and Webmail services and delay-tolerant Mapreduce workload. We scale the data to our assumed datacenter by removing demand peaks above $P_{grid}$. The traces are shown in Fig. 5, which shows peaks and variances, suggesting that SmartDPSS can help.

**System Parameters:** According to the recent empirical experiments, we assume that the limits of UPS charging/discharging rates are $B_{d, \text{max}} = B_{c, \text{max}} = 0.5\, \text{MW}$, and charging/discharging cost is $C_{b} = 0.1\, \text{dollars} [16]$. The maximum number of UPS charge/discharge cycles is $C_{\text{cycle}} = 5,000$ with a 4-year lifetime constraint [18]. The efficiency of UPS charging/discharging is $\eta_{c} = 0.8, \eta_{d} = 1.25$ [17]. We set the grid power limit as $P_{\text{grid}} = 2\, \text{MW}$ [16]. For simplicity, we use $B_{\text{max}} = 0, 15, 30$ to represent that battery capacity $B_{\text{max}}$ can power peak datacenter demand for 0, 15, 30 minutes.

**Compared Algorithms:** We compare SmartDPSS with the offline optimal algorithm and an online algorithm Impatient that always schedules workloads immediately regardless of the changes of electricity prices and renewable production.

**B. Analyzing Algorithm Sensitivity on Critical Factors**

From Theorem 2, we note that the performance of SmartDPSS depends on parameters $V, T, \epsilon$, battery capacity $B_{\text{max}}$ and two-timescale grid markets. We conduct sensitivity analysis on these critical factors to characterize their impact on DPSS operation cost and demand service delay.

1) **Impact of Control Parameter $V$:** As shown in Fig. 6(a) and 6(b), to simulate the day-ahead power market, we fix $T = 24$ time slots (one day). We conduct experiments with different values of $V$ with $\epsilon = 0.5, B_{\text{max}} = 15$ minutes. When $V$ increases from 0.05 to 5, SmartDPSS achieves a time-average cost that is closer to the optimal minimum value, while the average delay almost increases linearly. This quantitatively confirms the $[O(1/V), O(V)]$ cost-delay tradeoff in Theorem 2. Although Impatient algorithm indeed has lower delay, SmartDPSS can reduce significant cost with acceptable delay by choosing appropriate value of $V$, e.g., $V = 1$.

2) **Impact of Long-term Time Slot $T$:** In Fig. 6(c) and 6(d), we fix $V = 1, \epsilon = 0.5, B_{\text{max}} = 15$, and vary $T$ from 3 time slots (3 hours) to 144 time slots (6 days), which is a sufficient range for exploring the characteristics of different timescales of the long-term-ahead market. We observe that $T$ has relatively less impact on the cost of operating the DPSS. Although Theorem 3 shows that the impact of uncertain power demand and renewable energy increases proportionally with $T$, uncertainties can be alleviated by using the battery and two timescales power procurement. Thus, the time-average cost only fluctuates within $[-3.65\%, +6.23\%]$. This corroborates Theorem 3 that, even with infrequent actions of the DPSS operations, SmartDPSS can still achieve significant cost reduction. We also observe that average delay decreases with the increase of $T$, which verifies Theorem 3 that the queue backlog and delay is proportional to $1/T$. The rational is that with more frequent (smaller $T$) power management, the power demand is easier to meet (less delay).

3) **Impact of $\epsilon$, $B_{\text{max}}$ and two-timescale markets:** In Fig. 7, we compare the time-average total cost under different $\epsilon \in \{0.25, 0.5, 1, 2\}$ over the 31-day period with $V = 1, T = 24, B_{\text{max}} = 15$ and two-timescale markets. We can see that the cost increases as $\epsilon$ increases. However, from the definition of $\epsilon$, we know that the waiting time decreases with the increase of $\epsilon$. The rationale is that with larger $\epsilon$, the delay-aware queue $Y(t)$ grows faster. When minimizing the square of $Y(t)$ in Lyapunov optimization, more weight is given to delay control. So the demand can be queued shorter. However, if the demand delay is larger, it is easier to leverage future renewable energy and low energy prices, and hence can reduce more cost.

As shown in Fig. 7, we configure two cases to study the impact of grid markets: two-timescale markets and solely real-time market both with $V = 1, T = 24, B_{\text{max}} = 15, \epsilon = 0.5$. We can observe that the long-term-ahead market can bring in additional cost reduction. The reason is that not only can DPSS purchase certain energy beforehand in the long-term-ahead market with relative cheaper price, but also it is more flexible to take real-time management such as real-time purchasing or UPS charging/discharging to handle system uncertainties.

In Fig. 7, we also compare the time-average total cost under different battery sizes ($B_{\text{max}} \in \{0, 15, 30\}$) with $V = 1, T = 24, \epsilon = 0.5$ and two-timescale markets. It shows that the time-average total cost decreases with the increase of the UPS battery size. The rationale is that the UPS can offer additional free or cheaper power (stored superfluous renewable energy or purchased power when price is low) to serve the demand. In addition, we can obtain that the benefit brought by energy storage is higher than that of the markets structure, while the markets benefit is higher than that of parameter $\epsilon$. Thus, the optimal cost is mainly limited by the battery capacity.

**C. Characterizing Algorithm Robustness and Adaptivity**

As mentioned in Sec. IV, our SmartDPSS algorithm needs to approximate near future statistics as current values. Now we explore the influence of estimation errors on the performance of SmartDPSS. We add a random error to the datacenter power demand, solar energy production and energy prices, i.e., with
uniformly distributed ±50% errors [29]. We let SmartDPSS make all the control decisions based on the data set with random errors under different values of V. In Fig. 9, we show the differences in the percentage of DPSS operation cost reduction due to injected estimation errors, compared to the results we obtained using the original traces. We observe that the difference fluctuates within [−1.6%, 2.1%] for all values of V. Therefore, SmartDPSS is robust to our approximation.

Further, we study the impact of renewable energy penetration (the percentage of renewable energy in the total datacenter power supply) and datacenter power demand variation on the total cost. We simulate that the renewable energy penetration increases from 0 to 100%. We use the standard deviation to indicate the intensity of demand variation, i.e., \( \sqrt{\frac{1}{KT-1} \sum_{t=0}^{KT-1} [d(t) - E(d)]^2 \times p_d(t)} \), where \( E(d) \) is the expectation of the series of demand \( d(t) \) over time length \( t \in [0, KT] \), and \( p_d(t) \) is the distribution probability of \( d(t) \). We assume that the random variable of the datacenter power demand is uniformly distributed \( (p_d(t) = 1/KT) \). As expected, Fig. 8 shows that with the increase of penetration of renewable energy, the DPSS operation cost decreases significantly. The rational is that renewable energy is harvested cost-free (we do not consider the construction cost). In contrast, as the variation of demand increases, the operation cost increases slightly. The rational is that intensive variation incurs large approximation errors, making it difficult to take accurate power control decisions. But the UPS battery and two-timescale markets can deliver power continuously alleviating the power demand and supply fluctuations.

D. Verifying Algorithm Scalability

Based on the real-world traces in Fig. 5, we assume that the cloud service system expands to \( \beta \) times of the current datacenter power demand \( d(t) \) and renewable energy \( r(t) \). We evaluate the DPSS operation cost with \( \beta \in \{1, 2, 5, 10\} \) while fixing other parameters. Fig. 10 shows that the time-average total cost increases almost linearly (even logarithmically) with the expansion of \( d(t) \) and \( r(t) \). The increase rate slows down with the increase of \( d(t) \) and \( r(t) \). The main reason is that a large-scale system can provide more services and earn higher revenue, resulting in lower one-time construction and operation cost per unit service, e.g., cloud storage service offered by Google Drive.

VII. RELATED WORK

The first category of works is exploiting renewable energy in datacenters. Many large IT companies recently consider greening their datacenters with renewable energy [9]–[14]. However, the intermittent nature of renewable energy poses significant challenges to make use of them. Some works make the traffic “follow the renewables” to execute workload when/where renewable energy is available [9], [11]–[14]. However, these approaches require prediction of renewable energy production when scheduling workload or sacrifice performance to avoid wasting renewable energy. Other works supply renewable energy to deferrable loads to align demand with intermittent available renewable energy [23], [25], [26]. But they are from the prospective of renewable energy provider and do not consider the incorporation of energy storage and multiple markets in smart grid.

Another group of works is leveraging energy storage in datacenters. Recently, UPS shows its benefits to reduce electricity cost in datacenters [16], [18]–[21]. Datacenters can store energy in the UPS when energy prices are low and discharge UPS when prices are high, to reduce the power drained from grid [16], [21]. Moreover, UPS can shave peaks [18], [20]. During periods of low demand, UPS batteries store energy, while stored energy can be drained to temporarily augment the grid supply during emergencies of peak load. However, these works focus on studying the benefits of UPS battery for power cost reduction, and no renewable energy and demand management are considered. On the contrary, we leverage UPS to study how to manage power supply side and demand side of a datacenter in an integrated way.

Third stream of works is multiple timescale dispatch, pricing and scheduling in smart grid. Nair et al. [5] studied the optimal energy procurement from long-term, intermediate, and real-time markets under intermittent renewables. Jiang et al. [7] proposed optimal multi-period power procurement and demand response algorithm without energy storage. “Risk-limiting-dispatching” is proposed in [8] to manage integrated renewable energy. However, the above three approaches assume that the demand can be known ahead. Jiang et al. [6] solve the optimal day-ahead procurement and real-time demand response using dynamic programming, while He et al. [4] formulated the multi-timescale power dispatch and scheduling problem as a Markov decision problem. But both these approaches need substantial system statistics and are computationally expensive. We mitigate these disadvantages by applying two-stage Lyapunov optimization that makes online control decisions without a priori knowledge or any stationary distribution of energy prices, demand and supply.

In addition, interest has been growing in power management in smart grid and datacenters using Lyapunov optimization [27], [28]. In smart grid, several works have proposed...
optimal power management based on single-stage Lyapunov optimization. However, they either focused on managing individual household demand [22] or didn’t consider the incorporation and interaction of renewable energy and energy storage [7], [17], [22], [23]. In contrast, we manage the uncertain datacenter demand and multi-source energy supply in a systematic control view using two-stage Lyapunov [29], [30] have used two-stage Lyapunov to design two-timescale algorithm and T-Step Lookahead algorithm, but both of them study how to schedule jobs or distribute requests in solely grid-powered geographical datacenters rather than how to supply complementary multi-source energy in an uncertain datacenter environment with arbitrary demand.

**VIII. CONCLUSION AND FUTURE WORK**

This paper applied a two-stage Lyapunov optimization to design an online control algorithm, SmartDPSS, which optimally schedules multisource energy supply to power a datacenter with arbitrary demand, in a cost minimizing fashion. Without requiring a priori knowledge of system statistics, SmartDPSS can deliver reliable energy to a datacenter while minimizing DPSS operation cost over the datacenter’s long run. Both mathematical analysis and trace-driven evaluations demonstrated its optimality, robustness and scalability. Especially, it can approach the offline optimal cost within a diminishing gap of $O(1/V)$ while guaranteeing datacenter availability and bounding the demand service delay by $O(V)$, which is mainly decided by the UPS battery capacity, long-term vs. real-time markets, delay control parameter and DPSS operation frequency. What the role of energy storage in DPSS is and how to make more efficient demand response algorithm deserve deeper study.

**REFERENCES**


**APPENDIX A**

**PROOF OF Lemma 1**

Proof: Suppose for a contradiction that the optimal solution $II$ contains $g_r(t) > 0$ and $p_r(t) > 0$ for some $\tau$. Then we can construct another optimal solution $II'$ where $g_r(t) = 0$ for $\forall \tau$. In the optimal case, since the workload demand, renewable energy generation and electricity prices are known ahead, the optimal solutions have the same control decisions about when and how much to charge and discharge UPS to store and supply energy. Thus, the battery operation cost of $II$ and $II'$ is the same, denoted as $C_{battery}$. Since $p_r(t) > p_l(t)$ and $g_r(t)p_r(t) = 0$, the cost function of the solutions $Cost(II) = \sum_{t=0}^{t-1} [g_{ef}(t)/T_0 + g_r(t)p_r(t)] + C_{battery} > Cost(II') = \sum_{t=0}^{t-1} g_{ef}(t)/T_0p_l(t) + C_{battery}$. This contradicts the assumption that $II$ is the optimum.

**APPENDIX B**

**PROOF OF Lemma 2**

Proof: At any slot $t$, the new power demand $d_{dt}(t) \geq 0$. We will prove that the demand is served on or before time $t + \lambda_{max}$. If not, there is a contradiction. During time slots $\tau \in t + 1, ..., t + \lambda_{max}$, we know $Q(\tau) > 0$, otherwise $d_{dt}(t)$ will be served before $\tau$. Thus, $1Q(\tau) > 0 = 1$. Based on Eq. (12), we obtain:

$$Y(\tau + 1) \geq Y(\tau) - s_{dt}(\tau) + \epsilon.$$

Summing the above over $\tau \in t + 1, ..., t + \lambda_{max}$ yields:

$$Y(t + \lambda_{max} + 1) - Y(t + 1) \geq \lambda_{max}\epsilon - \sum_{\tau = t + 1}^{t + \lambda_{max}} s_{dt}(\tau).$$

Using the fact that $Y(t + 1) \geq 0$ and $Y(t + \lambda_{max} + 1) \leq Y_{max}$, and rearranging the above, we get:

$$\sum_{\tau = t + 1}^{t + \lambda_{max}} s_{dt}(\tau) \geq \lambda_{max}\epsilon - Y_{max}.$$

Since the demand is served in first-come-first-service manner in our model and $Q(t + 1) \leq Q_{max}$, we know that $d_{dt}(t)$ will be served on or before time $t + \lambda_{max}$, whenever there are at least $Q_{max}$ units of energy served for the delay-tolerant demand during $\tau \in t + 1, ..., t + \lambda_{max}$. Since we have assumed that $d_{dt}(t)$ is served before $t + \lambda_{max}$, we must have $\sum_{\tau = t + 1}^{t + \lambda_{max}} s_{dt}(\tau) < Q_{max}$. Substituting it into the above inequation yields:

$$Q_{max} > \lambda_{max}\epsilon - Y_{max}.$$

Rearranging the above, we get $\lambda_{max} < (Q_{max} + Y_{max})/\epsilon$, contradicting the definition of $\lambda_{max}$ in (13).

**APPENDIX C**

**PROOF OF Theorem 1**

Proof: Let $t = kT(k \in \mathbb{Z}^+)$ and $\tau \in [t, t + T - 1]$. Using the fact that $(\max(x, 0))^2 \leq x^2$, squaring the queue update Eq. (2) yields:

$$Q^2(\tau + 1) \leq Q^2(\tau) + [s_{dt}(\tau) + d_{dt}(\tau)]^2 - 2Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)].$$

Using the fact that for any $x \geq 0, y \geq 0, (x - y)^2 \leq x^2 + y^2$, and $s_{dt}(\tau) \in [0, S_{dt}^{\max}], d_{dt}(\tau) \in [0, D_{dt}^{\max}]$, we obtain:

$$Q^2(\tau + 1) - Q^2(\tau) \leq S_{dt}^{\max} + D_{dt}^{\max} - 2Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)].$$

Similarly, for virtual queues $X(\tau)$ and $Y(\tau)$, using $b_{rc}(\tau) \in [0, B_{rc}^{\max}], b_{dc}(\tau) \in [0, B_{dc}^{\max}]$, we have:

$$X^2(\tau + 1) - X^2(\tau) \leq B_{rc}^{\max} + B_{dc}^{\max} + 2X(\tau)[b_{rc}(\tau) - b_{dc}(\tau)],$$

$$Y^2(\tau + 1) - Y^2(\tau) \leq \epsilon^2 + S_{dt}^{\max} + 2Y(\tau)[\epsilon - s_{dt}(\tau)].$$

Now multiplying the above three inequations by 1/2, and summing them together, and taking expectations over $Q(\tau), X(\tau)$ and $Y(\tau)$, conditioning on $\Theta(\tau)$, we get the 1-slot conditional Lyapunov drift $\Delta_1 (\Theta(t))$:

$$\Delta_1 (\Theta(t)) \leq H_1 - E\{Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)]|\Theta(t)\} + E\{X(\tau)[b_{rc}(\tau) - b_{dc}(\tau)]|\Theta(t)\} + E\{Y(\tau)[\epsilon - s_{dt}(\tau)]|\Theta(t)\}.$$

Where $H_1 = S_{dt}^{\max} + \frac{1}{2}(D_{dt}^{\max} + B_{dc}^{\max} + B_{dc}^{\max} + \epsilon^2$.

Summing the above inequation over $\tau \in [t, t + 1, ..., t + T - 1]$, using the definition of $\delta Y(\Theta(t))$, we get an upper bound on $\Delta T (\Theta(t))$ as follows:

$$\Delta_1 (\Theta(t)) \leq E\{ \sum_{t+T}^{t+T-1} Y(\tau)[\epsilon - s_{dt}(\tau)]\}$$

$$+ H_1T - E\{ \sum_{t+T}^{t+T-1} Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)]\}$$

$$+ E\{ \sum_{t+T}^{t+T-1} X(\tau)[b_{rc}(\tau) - b_{dc}(\tau)]\}$$

Now adding to both sides the DPSS operation cost over the time slot $T$, i.e., the term $E\{ \sum_{t+T}^{t+T-1} Cost(\tau)|\Theta(t)\}$, we prove the theorem.

**APPENDIX D**

**PROOF OF Corollary 1**

Proof: According to the Eq. (2)(12)(15), for any $\tau \in [t, t + T - 1]$, we can get:

$$Q(t) - (\tau - t)S_{dt}^{\max} \leq Q(\tau) \leq Q(t) + (\tau - t)D_{dt}^{\max},$$

$$X(t) - (\tau - t)B_{max} \leq X(\tau) \leq X(t) + (\tau - t)B_{max},$$

$$Y(t) - (\tau - t)S_{dt}^{\max} \leq Y(\tau) \leq Y(t) + (\tau - t)\epsilon,$$

Therefore, recalling each terms in Eq. (19), we have:

$$\sum_{t+T}^{t+T-1} Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)]$$

$$\geq \sum_{t+T}^{t+T-1} [Q(t) - (\tau - t)S_{dt}^{\max}]\sum_{t+T}^{t+T-1} Q(\tau)[s_{dt}(\tau) + d_{dt}(\tau)]$$

$$\geq \sum_{t+T}^{t+T-1} [Q(t)s_{dt}(\tau) + d_{dt}(\tau)].$$
Similarly, we can also get:
\[
\sum_{\tau=t}^{t+T-1} X(\tau)[b_{rc}(\tau) - b_{dc}(\tau)]
\]
\[
\leq \sum_{\tau=t}^{t+T-1} [X(t) + (\tau - t)B^c_{max}][b_{rc}(\tau) - b_{dc}(\tau)]
\]
\[
\leq \sum_{\tau=t}^{t+T-1} X(t)b_{rc}(\tau) - b_{dc}(\tau) + T(T - 1)B^c_{max}^2,
\]
\[
\sum_{\tau=t}^{t+T-1} Y(\tau)[\epsilon - s_{dt}(\tau)]
\]
\[
\leq \sum_{\tau=t}^{t+T-1} [Y(t) + (\tau - t)\epsilon][\epsilon - s_{dt}(\tau)]
\]
\[
\leq \sum_{\tau=t}^{t+T-1} Y(t)[\epsilon - s_{dt}(\tau)] + T(T - 1)\epsilon^2.
\]
Therefore, by defining \( H_2 = H_1 + T(T - 1)B^c_{max}^2 + T(T - 1)\epsilon^2 \), using (19), we obtain:
\[
\Delta_T (\Theta(t)) \leq H_2 T + \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} Y(\tau)[\epsilon - s_{dt}(\tau)]|\Theta(t) \right\}
\]
\[
- \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} Q(t)[s_{dt}(\tau) + d_{dt}(\tau)]|\Theta(t) \right\}
\]
\[
+ \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} X(t)b_{rc}(\tau) - b_{dc}(\tau)\right\} |\Theta(t) \right\}.
\]
Adding \( V\mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} \text{Cost}(\tau)\right\} |\Theta(t) \right\} \) to both sides of the above inequality, we prove the theorem. \[\blacksquare\]

**APPENDIX E**

**PROOF OF LEMMA 3**

**Proof:** The proof is similar to that of Lemma 5 in the work of UPS battery management in datacenter [16]. \[\blacksquare\]

**APPENDIX F**

**PROOF OF THEOREM 2**

**Proof:** (1) From Lemma 3, we know that the optimal solution has the following properties: if \( X(t) > 0 \), then \( b_{rc}(t) = 0 \); if \( X(t) < -[Q(t) + Y(t)]_{max} = -U_{max} \), then \( b_{dc}(t) = 0 \). We prove the result using induction. Since \( X(0) = b(t) - U_{max} - B_{min} - B^d_{max}, B_{min} \leq b(0) \leq B_{max} \), we know that \( -U_{max} - B^d_{max} \geq X(0) \leq -U_{max} - B_{min} - B^d_{max} \).

Now we first consider \( 0 < X(t) \leq B_{max} - U_{max} - B_{min} - B^d_{max}, \) then \( b_{rc}(t) = 0 \). Since there is no battery recharging and the maximum discharged energy is \( B^d_{max} \) each time, we have:
\[
-U_{max} - B^d_{max} < -B^d_{max} < X(t + 1) \leq X(t) \leq B_{max} - U_{max} - B_{min} - B^d_{max}.
\]
Next, suppose \( -V_{P_{max}}/T < X(t) \leq 0 \), then \( b_{dc}(t) = 0 \). The maximum charging and recharging energy at each time are \( B_{max} \) and \( B_{max}^c \) respectively. Thus we have:
\[
-U_{max} - B^d_{max} < -V_{P_{max}}/T - B^d_{max} \leq X(t + 1) \leq X(t) + B_{max} \leq B_{max} - U_{max} - B_{min} - B^d_{max} \text{ (using (21)).}
\]
Next, suppose \( -U_{max} < X(t) \leq -V_{P_{max}}/T. \) Similarly, we have:
\[
-U_{max} - B^d_{max} \leq X(t) - B^d_{max} \leq X(t + 1) \leq X(t) + B_{max} - V_{P_{max}}/T + B^c_{max} \text{ (using (21)).}
\]
According to the definition of \( V_{max} \), we have:
\[
V_{P_{max}}/T \leq B_{max} - B_{min} - B^d_{max} - D_{dt}^d + \epsilon.
\]
Then we have:
\[
X(t + 1) \leq -V_{P_{max}}/T + B^c_{max} \leq B_{max} - U_{max} - B_{min} - B^d_{max} (U_{max} = V_{P_{max}}/T + D_{max}^d + \epsilon).
\]
Finally, we consider the case of \( -U_{max} - B^d_{max} < X(t) \leq -U_{max} \). Since \( x(t) < -U_{max}, b_{dc}(t) = 0 \). Then \( -U_{max} - B^d_{max} \leq X(t) \leq X(t + 1) \leq -U_{max} \). We now complete the proof.

(2) From Eq. 14 and inequality 21, we have:
\[
-U_{max} - B_{max} \leq X(t) = b(t) - U_{max} - B_{min} - B_{max} \leq B_{max} - U_{max} - B_{min} - B^d_{max}.
\]
It is easy to see that \( B_{min} \leq b(t) \leq B_{max} \).

(3) We prove \( Q(t) < Q_{max} \) using induction. Obviously, \( Q(0) = 0 < Q_{max} \). Now we assume that \( Q(t) < Q_{max} \), then \( Q(t + 1) < Q_{max} \). If \( Q(t + 1) \leq V_{P_{max}}/T + D_{max}^d \), then the deferred workload can increase by at most \( D_{max}^d \) on slot (see (2.2)).

Next we consider the case when \( V_{P_{max}}/T < Q(t) \leq V_{P_{max}}/T + D_{max}^d \). Consider the term involving \( g_{ws}(t) \) in problem (4): \( g_{ws}(t)[V_{P_{ws}}(t)/T - Q(t) - Y(t)] \). Then, \( Q(t) + Y(t) \geq Q(t) > V_{P_{max}}/T \geq V_{P_{ws}}(t)/T \) (because \( T \geq 1 \), and our algorithm will choose \( g_{ws}(t) = P_{grid} \)).

If \( Q(t) - s_{max}^d > 0 \), then at time slot \( t \), DPSS serves at least \( s_{max}^d \) delay-tolerant power demand. Since the newly deferred workload is at most \( D_{max}^d \) and \( s_{max}^d \geq D_{max}^d \), the queue \( Q(t) \) will not increase. Thus, \( Q(t + 1) < Q(t) \leq V_{P_{max}}/T + D_{max}^d \).

If \( Q(t) - s_{max}^d \leq 0 \), then \( Q(t + 1) = d_{dt}(t) \leq D_{max}^d \).

The proof of \( Y_{max} \leq V_{P_{max}}/T + \epsilon \) and \( Q(t) + Y(t) \leq U_{max} \) for all \( t \) are similar and omitted for brevity.

(4) This follows straightforward from Lemma 2 together with part (3).

(5) Let \( t = kT (k \in \mathbb{Z}^+) \) and \( \tau \in [t, t + T - 1] \). From Lemma 4, we know that in the optimal solution \( \phi^{opt} \), \( \mathbb{E}\{b_{rc}(\tau) - b_{dc}(\tau)\} = 0, \mathbb{E}\{\epsilon - s_{dt}(\tau)\} = 0 \). Since SmartDPSS minimizes the RHS of Eq. (20), plugging the policy \( \pi \) into the RHS of Eq. (20), we have:
\[
\Delta (\Theta(t)) + V\mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} \text{Cost}_{av}(\tau)|\Theta(t) \right\} \leq H_2 T + V\mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} \text{Cost}(\tau)|\Theta(t) \right\} - \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} Q(t)[s_{dt}(\tau) + d_{dt}(\tau)]|\Theta(t) \right\} + \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} Y(t)[\epsilon - s_{dt}(\tau)]|\Theta(t) \right\} \leq H_2 T + V\phi^{opt}.
\]
Taking the expectation of both sides and rearranging the term, we get:
\[
V\mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} \text{Cost}_{av}(\tau) \right\} + E[L(\Theta(t + T)) - L(\Theta(t))] \leq H_2 T + V\phi^{opt}.
\]
Summing the above over \( t = kT, k = 0, 1, 2, ..., K - 1 \), using the fact that \( L(\Theta(t)) > 0 \), dividing both sides by \( V KT \), we obtain:
\[
\frac{1}{K T} \mathbb{E}\left\{ \sum_{\tau=0}^{K T-1} \text{Cost}_{av}(\tau) \right\} \leq \phi^{opt} + \frac{H_2}{V}.
\]
Taking limit as \( K \to \infty \), we complete the proof. \[\blacksquare\]

(6) Since SmartDPSS makes decisions to satisfy all the constraints in problem (4) and (5), combining the constraints
together, all the constraints of problem \( P1 \) are satisfied. Therefore, SmartDPSS control decisions are feasible to problem \( P1 \).

**APPENDIX G**

**PROOF OF THEOREM 3**

*Proof:* When using the data \( \hat{Q}(t), \hat{X}(t), \hat{Y}(t) \) to carry out our algorithm, we still try to minimize the RHS of (20). Denote \( e^Q(t) = \hat{Q}(t) - Q(t), e^X(t) = \hat{X}(t) - X(t) \) and \( e^Y(t) = \hat{Y}(t) - Y(t) \). We define the objective as \( f(\Theta(t)) \):

\[
f(\Theta(t)) \triangleq V E\{\sum_{t=T}^{t+T-1} Cost(\tau)|\Theta(t)\}
- E\{\sum_{t=T}^{t+T-1} \hat{Q}(t)[s_{dt}(\tau) + d_{at}(\tau)]|\Theta(t)\}
+ E\{\sum_{t=T}^{t+T-1} \hat{X}(t)[b_{rc}(\tau) - b_{dc}(\tau)]|\Theta(t)\}
+ E\{\sum_{t=T}^{t+T-1} \hat{Y}(t)[\epsilon - s_{dt}(\tau)]|\Theta(t)\}
= V E\{\sum_{t=T}^{t+T-1} Cost(\tau)|\Theta(t)\}
- E\{\sum_{t=T}^{t+T-1} e^Q(t)[s_{dt}(\tau) + d_{at}(\tau)]|\Theta(t)\}
+ E\{\sum_{t=T}^{t+T-1} e^X(t)[b_{rc}(\tau) - b_{dc}(\tau)]|\Theta(t)\}
+ E\{\sum_{t=T}^{t+T-1} e^Y(t)[\epsilon - s_{dt}(\tau)]|\Theta(t)\}
\]

Denote the minimum value of \( f(\Theta(t)) \) and \( f(\Theta(t)) \) as \( \hat{f} \) and \( f^* \), respectively. Using the fact that \( |a-b| \leq |a|+|b| \), we know \( |b_{rc}(\tau) - b_{dc}(\tau)| \leq B_{rc}^{max} + B_{dc}^{max}, |\epsilon - s_{dt}(\tau)| \leq \epsilon + S_{dt}^{max} \). Hence, we have:

\[
\hat{f} \leq f^* + T \theta_{max}[2S_{max}^{dt} + \epsilon + B_{max}^{rc} + B_{max}^{dc} + D_{max}^{dt}].
\]

This shows that Eq. (20) holds with \( Q(t), X(t), Y(t) \) replaced by \( \hat{Q}(t), \hat{X}(t), \hat{Y}(t) \), and \( H_2 \) replaced by \( H_3 = H_2 + T \theta_{max}[2S_{max}^{dt} + \epsilon + B_{max}^{rc} + B_{max}^{dc} + D_{max}^{dt}] \). The rest of the proof is similar to the proof of Eq. (27) in Theorem 2. \( \blacksquare \)